Unspanned Macroeconomic Factors in Oil Futures

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Abstract

This paper constructs a macro-finance model for commodity futures, and documents a new empirical fact that real economic activity forecasts oil futures returns and prices. The model generalizes previous futures pricing models and indicates a time-varying oil risk premium that covaries strongly with the business cycle. Model estimates reveal rich dynamics between economic activity and oil futures markets which are material to the valuation of real options.

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1 Introduction

Research in finance has yielded models that closely fit the dynamics of futures markets. However, these models explain prices and expected returns in terms of latent state variables with no clear relation to the real economy.[1] On the other hand, macroeconomic research has investigated the dynamics of oil prices with economic data, finding that oil shocks forecast recessions.[2] However, these studies use spot prices only, while futures markets are more active than spot markets and have a term structure that can tell us about risk premia and the market's forecast of the spot price.

This paper examines the interactions between futures markets and the real economy and documents a new empirical fact: real economic activity forecasts prices and returns in oil futures markets. Previous research documents that the price of oil affects the real economy, while this paper is the first to show a converse effect flowing from the real economy to the futures market and to examine the resulting feedback relationship.

I construct a futures pricing model with both latent and macroeconomic factors which makes two contributions to the literature. First, it allows us to estimate the joint dynamics of macro factors with futures prices and returns in a consistent way. To my knowledge this is the first such macro-finance model for commodity futures. Second, it extends and generalizes previous models in a way that is key to the empirical findings. While previous models do not explicitly examine how futures markets interact with macro factors, they implicitly impose strong restrictions on those interactions. Specifically, they assume that all relevant information is fully reflected in the term structure of futures prices (perfect risk spanning). I show this restriction is rejected by the data; various measures of real activity

[1] e.g. Gibson and Schwartz (1990); Schwartz (1997); Casassus and Collin-Dufresne (2005)

[2] e.g. Hamilton (1983); Bernanke et al. (1997); Hamilton (2003); Barsky and Kilian (2004); Kilian (2009)

forecast oil futures returns and prices, over and above the information in current futures prices.

As in the previous literature I concentrate on oil because oil is the single most important commodity to the economy as reflected by its input share, trading volume, media coverage and academic and industry attention. The Energy Information Administration (EIA) estimates that in 2010 expenditures on energy, the majority of which was petroleum based, accounted for 8.3% of U.S. GDP.[3] On the financial side, the oil futures market is the largest and most active commodity futures market. With the rise of commodity exchange traded funds (ETFs), index funds, and pension and hedge fund participation it increasingly represents a vital financial market in its own right; in 2014 the average daily trading volume in Nymex crude oil futures was $53 billion compared to an average daily trading volume of $41 billion on the entire New York Stock Exchange (NYSE).

Real economic activity is the most common macro factor used in macro-finance studies of other markets and should clearly be material to the price of oil, given oil's input share into the real economy. Previous research[4] finds that real activity forecasts countercylical returns in equity and bond markets. The discovery that real activity forecasts procyclical returns to oil futures echoes those findings but in the opposite direction. I argue the findings are consistent with oil futures as a hedge asset for consumers, while I do not find support for the classic hedging pressure hypothesis.

Studies such as Casassus and Collin-Dufresne (2005) and Hamilton and Wu (2014) imply that the oil risk premium is stable over time and does not covary with the business cycle. This is a consequence of the spanning assumption plus the fact that the contemporaneous

[3]The EIA's figure reflects expenditures on combustible petroleum products and does not include products such as asphalt, tar, wax, coke, lubricants, and petrochemicals that are critical inputs into many industries.

[4] See, e.g., Chen, Roll and Ross (1986), Cooper and Priestley (2008), Ludvigson and Ng (2009), Joslin, Priebsch and Singleton (2014).

correlation of oil prices with the business cycle is low. Relaxing this restriction produces an estimated oil risk premium that better explains the historical returns to oil futures, more than tripling the adjusted R^2 from 1.3% to 4.3%. The implied monthly risk premium is nine times more volatile than that implied by the restricted model, and covaries strongly with the business cycle. These results indicate that previous models fail to capture the majority of time variation in the oil risk premium.

Pindyck (1993) argues that the value of real options should depend on macro in addition to financial factors. However, this has not been emphasized in the literature on commodity real options (i.e. optimal resource extraction) because of the assumption of perfect risk spanning, which rules out the relevance of macro factors. In a calibrated example, I find that adding real activity into the model increases the value of a hypothetical oil well. When the payoff to a real option depends on a macro factor there are two channels by which the macro factor affects option value: its unspanned dynamics and its risk premium. In the example I find the dynamics effect dominates while the risk premium effect is small.

The model developed in this paper makes use of the full panel of futures prices and macroeconomic time series and reveals rich dynamics that are not available from vector autoregressions. For example, a higher oil price forecasts lower real activity consistent with the evidence that oil shocks forecast recessions (Hamilton, 1983; Bernanke et al., 1997; Kilian, 2009). I find that this well-documented effect varies depending on the market's forecast of oil prices: oil shocks that are forecast by the market to be more persistent have a larger and longer lasting effect on real activity. Conversely, although shocks to real activity dissipate in less than a year the market forecasts that the resulting higher oil price persists for decades, perhaps because oil is a nonrenewable resource. I also find that the effects of oil shocks are stronger when the economy is in expansion, and that real activity drives oil prices through shocks to industrial production while oil shocks drive real activity through changes in consumer spending.

1.1 Related Literature

There are two strands of the literature in commodity futures that this paper builds upon. In the first, commodity futures prices are modeled as affine functions of latent (unobserved) state variables. Classic examples are Gibson and Schwartz (1990), Schwartz (1997), and Casassus and Collin-Dufresne (2005); more recent examples include Casassus, Liu and Tang (2013) and Hamilton and Wu (2014). Models of this type do not incorporate macroeconomic data and the latent variables can often be rotated and translated without changing the likelihood (Dai and Singleton (2000); Collin-Dufresne, Goldstein and Jones (2008)), so their economic meaning is unclear. More subtly, they implicitly assume that all relevant information in the economy is reflected (spanned) in current futures prices and no other information can contribute incremental forecasting power. I find that real economic activity has material effects on risk premiums and forecasts of oil futures prices, over and above the information in current futures prices.

The second strand uses vector autoregressions (VARs) to explore the time series relation of oil prices with the real economy; examples include Hamilton (1983); Hamilton (2003); Kilian (2009); Alquist and Kilian (2010); Kilian and Vega (2011). These studies include a single state variable based on the spot price of oil. This approach does not incorporate the full panel of futures prices of different maturities, and is silent regarding risk premia and the market's forecast of the spot price. The model in this paper imposes the additional assumption that risk premiums are "essentially affine" (Duffee, 2002) in the state variables, which lets us bring the full term structure of futures prices to bear on returns, forecasts, and risk spanning.

Fama and French (1987), Bessembinder and Chan (1992), Singleton (2013) and Hamilton and Wu (2015) run return forecasting regressions for individual returns to futures on individual commodities; Gorton, Hayashi and Rouwenhorst (2013) sort the cross section of commodities into portfolios. Szymanowska et al. (2013) decompose individual futures returns into a spot premium and a term premium. Hong and Yogo (2012) find that futures market open interest has robust predictive power for futures returns and bond returns. This paper contributes to this literature as well, as it offers a simple and consistent way to make use of the full term structure of futures prices and returns.

Chiang, Hughen and Sagi (2015) extract spanned factors from oil futures and a volatility factor from oil options, and find that exposure to the volatility factor carries a risk premium in equity markets but not in oil futures. In contrast, I examine the effects of the real economy on price forecasts and risk premia in oil futures markets, and the resulting dynamic feedback relationship, in a macro-finance setting.

2 Data and Forecasting Regressions

In this section I describe the data and investigate to what extent the macroeconomic time series are spanned by oil futures prices. The distinction between spanned and unspanned macro factors drives the modelling strategy. To see the distinction, let R_{t+j} be the payoff from going long a $j-$period futures contract at price F_t^j and holding it to maturity:

$$R_{t+j} = S_{t+j} - F_t^j$$

where S_{t+j} is the spot price at maturity. This accounting identity always holds ex post and thus holds in expectation for any information set X_t :

$$F_t^j = E\left[S_{t+j}|X_t\right] - E\left[R_{t+j}|X_t\right]$$

This is the case for the spanning assumption: any information relevant to forecasting spot prices or returns should be reflected in contemporaneous futures prices. However, the same argument applies to bond yields and bond returns, yet recent research finds evidence against it in bond markets. In particular, real activity (Joslin, Priebsch and Singleton (2014)) or latent factors extracted by filtering (Duffee (2011)) or from forecasts (Chernov and Mueller (2012)) help to forecast bond returns over and above information in the term structure of bond yields. The first contribution of this paper is to document similar evidence against the spanning assumption in oil futures markets.

2.1 Futures Price Data

I use closing prices for West Texas Intermediate (WTI) oil futures with maturities of one to twelve months, on the last business day of each month from January 1986 to June 2014. The futures price data is denoted

$$f_t^j = \log(F_t^j),\ j = 1...J,\ \ t = 1...T$$

$$f_t = \left[\begin{array}{cccc} f_t^1 & f_t^2 & ... & f_t^J \end{array}\right]'$$

where F_t^j is the closing price at end of month t of the future that expires in month $t + j$, $t = 1$ corresponds to 1/1986, $T = 342$ corresponds to 6/2014, and $J = 12$. The maximum maturity of twelve months is because longer dated futures were seldom traded in the early years of the sample. The results do not change significantly if I extend J to, e.g., 24 months maturity.

2.2 Macro Factors

I use the Chicago Fed National Activity Index (CFNAI), hereafter labelled GRO, as a forward-looking measure of real economic activity. The CFNAI is a weighted average of 85 U.S. macroeconomic time series, published monthly by the Chicago Fed.[5]

Measures of real activity have been found to forecast inflation (Stock and Watson, 1999), bond returns (Ludvigson and Ng, 2009) and equity returns (Cooper and Priestley (2008)) and are commonly used in macro-finance studies of the term structure (Ang and Piazzesi 2003; Joslin, Priebsch and Singleton 2014). Importantly, the results in this paper do not depend on using the CFNAI measure specifically but also obtain using other indexes of real activity (see Appendix A).

The second macro factor is the inventory of oil in readily available storage. The Theory of Storage (Working (1949)) predicts a natural equilibrium between the cost of carry in the futures market and the level of inventories held. Also, Gorton, Hayashi and Rouwenhorst (2013) find that inventories are associated with returns to futures contracts, using portfolio sorts of the cross-section of commodity futures. I use the log of the Energy Information Administration's "Total Stocks of Commercial Crude Oil excluding the Strategic Petroleum Reserve" as a measure of the available inventory of crude oil,[6] hereafter labelled INV.

Thus, the macro factors are $M_t = [GRO_t, INV_t]'$. Figure 1 plots the time series of log oil futures prices and the macro factors GRO and INV.

[5] https://www.chicagofed.org/publications/cfnai/index

[6] http://www.eia.gov/petroleum/

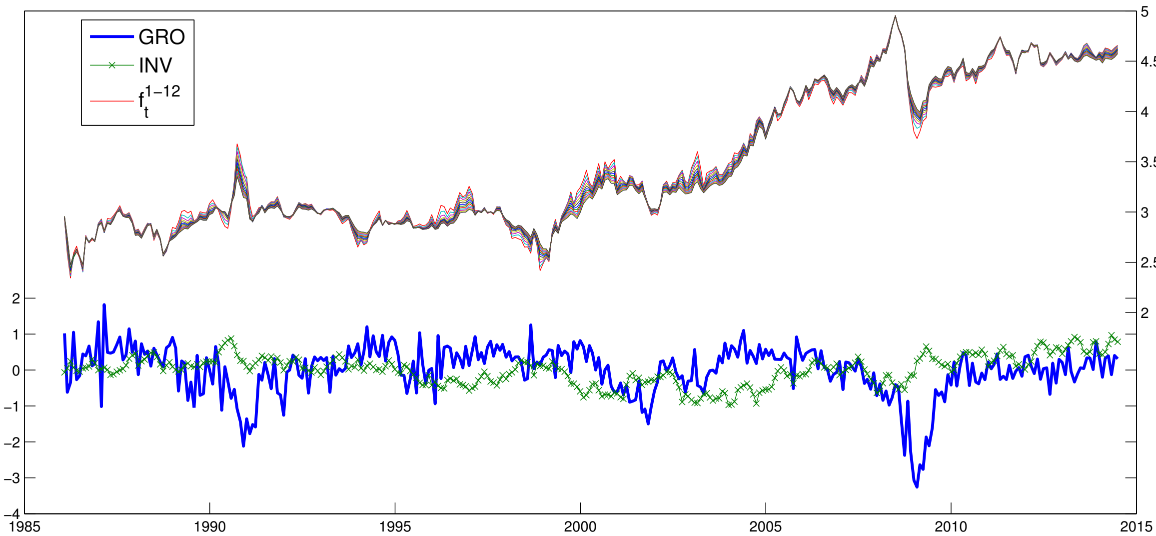

Figure 1: The figure plots log futures prices for Nymex crude oil f_t^{1-12}, the Chicago Fed National Activity Index GRO, and the log of the EIA's monthly U.S. oil inventory INV from January 1986 to June 2014.

2.3 Stylized Facts

Previous affine futures pricing models (e.g. Gibson and Schwartz (1990); Schwartz (1997); Casassus and Collin-Dufresne (2005)) assume that all relevant factors are spanned by the futures market and hence they are estimated using futures prices only. As Duffee (2011) and Joslin, Le and Singleton (2013) observe in the context of bond yields, the spanning assumption has strong implications for the joint behavior of futures prices and the economy. First, any such model with N state variables can be rotated into "reduced form" such that the reduced-form state variables are equal to the prices of N arbitrary linearly independent portfolios of futures contracts (Duffie and Kan (1996)). Second, the N portfolios explain log futures prices up to idiosyncratic errors. Third, conditional on the prices of the N portfolios, no other information can contribute incremental forecasting power.

I document three stylized facts that contradict these restrictions. First, the first two

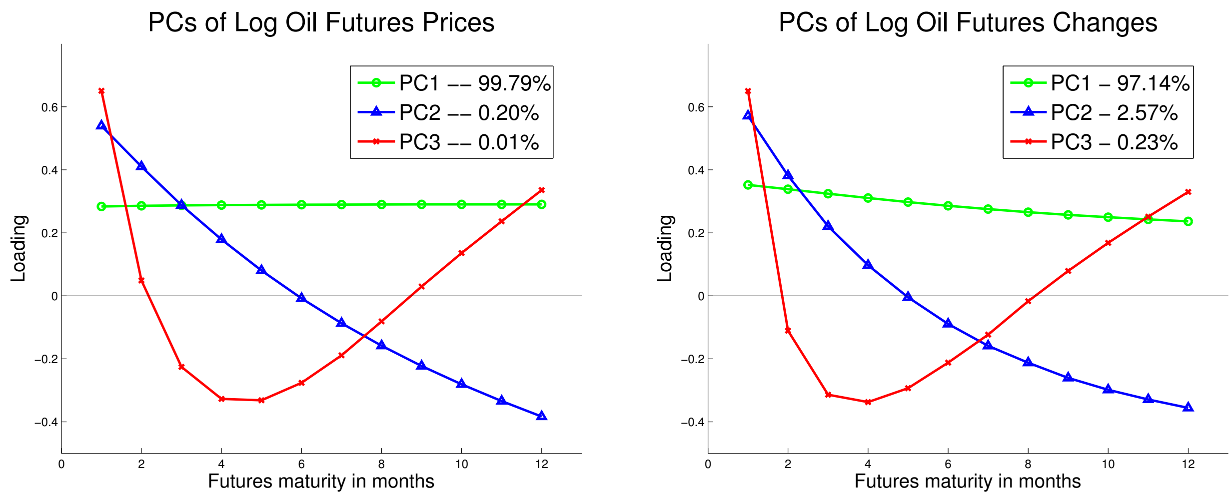

Figure 2: Loadings of the first three principal components (PCs) of the levels (panel A) and changes (panel B) of log oil futures prices, monthly from 1/1986 - 7/2014. The legend displays the fraction of total variance explained by the PCs.

principal components account for more than 99% of the variation in levels and changes of log oil futures prices. There are more than two sources of economic uncertainty in the world, so the natural hypothesis is that some relevant economic information may be unspanned by oil futures. Second, I find that the macro factors M_t, in particular GRO, are not well summarized by futures prices. Third, I find that M_t contributes incremental forecasting power for oil prices and returns over and above the information in contemporaneous futures prices.

A) Oil futures prices display a low dimensional factor structure

Figure 2 plots the loadings of the first three principal components (PCs) for the levels and changes in log oil futures prices by maturity. The first two PCs have the familiar level and slope patterns of loadings, and account for 99.9% of the variation in log price levels and 99.7% of variation in log price changes.

B) M_t is mostly unspanned by oil futures

I project M_t on the first two principal components of log oil futures prices and label the residual UM_t:

$$M_t = \alpha + \gamma_{1,2} PC_t^{1,2} + UM_t$$

The R^2 of the projections for GRO and INV are 6.4% and 27.5% . Projecting on the first five PCs the R^2 are 14.5% and 30.0% and projecting on all 12 individual futures maturities the R^2 are 18.9% and 30.9%. Thus, much of the monthly variation in M_t – and particularly GRO – is unspanned by variation in oil futures prices. However, M_t might be measured with error or some subcomponents of M_t may be irrelevant to oil prices and risk premia. Thus the main question is not the projection R^2 but whether M_t is economically relevant over and above the information in futures prices.

C) M_t forecasts returns over and above information in the futures curve

Table 1 Panel A shows the results of forecasting returns to oil futures using information from the current futures curve and then adding the macro variables M_t. We see that M_t – particularly GRO – contributes additional forecasting power, over and above the information in the futures curve, for both returns to the second nearby contract (columns 1-3) and average returns to all traded futures that month, which is effectively the same as the return to the level portfolio (columns 4-6). In the same way, Panel B shows that M_t forecasts changes in the level, but not the slope. The coefficients on f^{1-12} in these regressions (not shown) have different signs on adjacent maturities, a clear sign of overfitting, yet M_t still contributes substantial forecasting power.

The forecasting power of GRO is economically significant. Focus on the columns where

Table 1: Panel A shows the results of forecasting the log return to the second nearby oil futures contract (r^2_{t+1}) and the average log return to all active oil futures with maturities from 2 to 12 months ($\bar{r}_{t+1}$). Panel B shows the results of forecasting changes in the principal components of log prices. GRO is the Chicago Fed National Activity Index and INV is the log of U.S. crude oil stocks. The data are monthly from from 1/1986 to 6/2014. Newey-West standard errors with six lags are in parentheses.

Panel A: Forecasting Returns

$$r_{t+1} = \alpha + \beta_{GRO,INV} M_t + \beta_{\mathcal{P}} \mathcal{P}_t + \epsilon_{t+1}$$

	r^2_{t+1}			$\bar{r}_{t+1}$		
GRO_t	0.0259**	0.0242**	0.0215*	0.0194**	0.0194**	0.0175*
	(0.0109)	(0.0112)	(0.0115)	(0.0092)	(0.0095)	(0.0097)
INV_t	0.040	0.037	0.035	0.018	0.009	0.007
	(0.092)	(0.093)	(0.093)	(0.076)	(0.075)	(0.076)
Spanned Factors $\mathcal{P}_t$:	$PC^{1,2}$	PC^{1-5}	f^{1-12}	$PC^{1,2}$	PC^{1-5}	f^{1-12}
T	341	341	341	341	341	341
Adj. $R^2(\mathcal{P}_t)$	0.4%	0.7%	4.6%	0.2%	0.1%	3.9%
Adj. $R^2(\mathcal{P}_t + M_t)$	3.3%	3.0%	6.2%	2.7%	2.3%	5.5%
F-ratio	6.1***	4.9***	3.9**	5.2***	4.7***	3.8**

Panel B: Forecasting PCs

$$\Delta PC_{t+1} = \alpha + \beta_{GRO,INV} M_t + \beta_{\mathcal{P}} \mathcal{P}_t + \epsilon_{t+1}$$

	ΔPC^1 (Change in level)			ΔPC^2 (Change in slope)		
GRO_t	0.0653**	0.0659**	0.0599*	0.0089**	0.0079*	0.0067
	(0.0315)	(0.0326)	(0.0331)	(0.0042)	(0.0045)	(0.0046)
INV_t	0.059	0.027	0.018	0.041	0.047	0.046
	(0.259)	(0.257)	(0.260)	(0.053)	(0.048)	(0.045)
Spanned Factors $\mathcal{P}_t$:	$PC^{1,2}$	PC^{1-5}	f^{1-12}	$PC^{1,2}$	PC^{1-5}	f^{1-12}
T	341	341	341	341	341	341
Adj. $R^2(\mathcal{P}_t)$	-0.4%	-0.5%	2.9%	6.5%	8.0%	10.3%
Adj. $R^2(\mathcal{P}_t + M_t)$	2.0%	1.6%	4.6%	7.5%	8.8%	10.8%
F-ratio	5.1***	4.7***	3.8**	1.3	1.1	0.8

the spanned factors are the first two PCs of log futures prices, which captures 99.99% of the variation in futures prices and also minimizes overfitting. The adjusted R^2s for returns and the level factor go from 0.4%, 0.2% and -0.4% to 3.3%, 2.7% and 2.0%. The point estimates are also economically large. A one percent increase in GRO, which is about two standard deviations, forecasts a return to the level portfolio of 194 basis points over the next month. However, GRO reverts quickly toward its mean, and oil futures are volatile – the monthly standard deviations of returns are 9.6% for r_t^2 and 7.7% for $\bar{r}_t$. As a result, the corresponding implied Sharpe ratios are modest, at 0.59 and 0.60 for the second nearby future and the level portfolio respectively. Thus, while real activity is a first-order determinant of the oil price forecast the level of implied predictability is not implausibly large.

These results are not driven by the sample period or the specific measure of real activity. Appendix A shows that the forecasting power of real activity for oil prices and returns is robust to using two alternative measures of real activity, excluding the volatile time period after 2007, including measures of time-varying volatility, and using year-on-year changes to eliminate persistent regressors.

3 Model

Motivated by the stylized facts described in Section 2, this section develops a macro-finance model for commodity futures that admits unspanned macroeconomic state variables. Let X_t be a vector of N state variables that summarize the economy. X_t includes macroeconomic risk factors such as expected economic growth, and factors specific to the commodity such as hedging pressure, inventories, and expectations of supply and demand. The state vector follows a Gaussian VAR,

$$X_{t+1} = K_{0X}^{\mathbb{P}} + K_{1X}^{\mathbb{P}} X_t + \Sigma_X \epsilon_{t+1}^{\mathbb{P}} \tag{1}$$

where $\epsilon_{t+1}^{\mathbb{P}} \sim N(0, 1_N)$. Risk premiums are "essentially affine" in the state variables (Duffee, 2002) so the stochastic discount factor is given by

$$\mathcal{M}_{t+1} = e^{(\Lambda_0 + \Lambda_1 X_t)' \epsilon_{t+1}} \tag{2}$$

This specification includes previous benchmark models such as Gibson and Schwartz (1990); Schwartz (1997); Casassus and Collin-Dufresne (2005). All of these previous models implicitly assume perfect spanning i.e. the state X_t is fully reflected in contemporaneous futures prices. As is well known for bond yields (Duffie and Kan, 1996), the spanning assumption implies that X_t can be replaced by an arbitrary set of linear combinations of log futures prices:

$$\mathcal{P}_t^N = W f_t$$

where W is *any* full rank $N \times J$ matrix. Thus, the spanning assumption implies:

1. Futures prices are described up to idiosyncratic errors by the N factors $\mathcal{P}_t^N$.
2. The projection of X_t on $\mathcal{P}_t^N$ has R^2 of one.
3. Conditional on $\mathcal{P}_t^N$, no other information forecasts X_t or futures prices or returns.

I instead assume that a subspace of X_t is spanned, while its complement is unspanned but observed by the econometrician. Suppose that contemporaneous futures prices are determined by a set of linear combinations $L_t = V f_t$ where V is a real valued $N_L \times J$ matrix and $N_L < N$. That is, the spot price and its evolution under the risk neutral measure are given by:

$$s_t = \delta_0 + \delta_1' L_t \tag{3}$$

$$L_{t+1} = K_{0L}^{\mathbb{Q}} + K_{1L}^{\mathbb{Q}} L_t + \Sigma_L \epsilon_{t+1}^{\mathbb{Q}} \tag{4}$$

where $\epsilon_{t+1}^{\mathbb{Q}} \sim N(0, 1_{N_L})$ and $\Sigma_L = V\Sigma_X$.[7] By the same rationale as before we can replace L_t with N_L linear combinations of log prices,

$$\mathcal{P}_t^L = W_L f_t$$

where W_L is any full rank $N_L \times J$ matrix, and transform the state space from X_t to $(\mathcal{P}_t^L, UM_t)$ where UM_t are the unspanned components (projection residuals) of the macro factors M_t. In contrast to the perfect-spanning models, this model implies that:

1. Futures prices are described up to idiosyncratic errors by $N_L < N$ factors.

2. The projection of X_t on $\mathcal{P}_t^L$ has R^2 less than one.

3. Conditional on $\mathcal{P}_t^L$, other information may forecast X_t or futures prices or returns.

Motivated by the variance decomposition in the previous section, I assume the number of spanned state variables $N_L = 2$. After estimating the model I rotate and translate so that the state variables correspond to the model implied spot price and cost of carry (s_t, c_t) and the macroeconomic series M_t.[8]

The model can then be described in just two equations:

1) the law of motion for the state variables:

[7] This specification is in the class of "hidden-factor" models explored by Diebold, Rudebusch and Aruoba (2006); Duffee (2011); Chernov and Mueller (2012); Joslin, Priebsch and Singleton (2014) for bond yields.

[8] A detailed description of the parametrization and estimation are in the Internet Appendix.

$$\begin{bmatrix} s_{t+1} \\ c_{t+1} \\ M_{t+1} \end{bmatrix} = \begin{bmatrix} K^{\mathbb{P}}_{0sc} \\ K^{\mathbb{P}}_{0M} \end{bmatrix} + \begin{bmatrix} K^{\mathbb{P}}_{sc,sc} & K^{\mathbb{P}}_{sc,M} \\ K^{\mathbb{P}}_{M,sc} & K^{\mathbb{P}}_{MM} \end{bmatrix} \begin{bmatrix} s_t \\ c_t \\ M_t \end{bmatrix} + \Sigma\epsilon^{\mathbb{P}}_{t+1} \tag{5}$$

2) the dynamics of (s_t, c_t) under the risk neutral measure:

$$\begin{bmatrix} s_{t+1} \\ c_{t+1} \end{bmatrix} = K^{\mathbb{Q}}_0 + K^{\mathbb{Q}}_1 \begin{bmatrix} s_t \\ c_t \end{bmatrix} + \Sigma_{sc}\epsilon^{\mathbb{Q}}_{t+1} \tag{6}$$

The model is a canonical form, that is, any affine futures pricing model with two spanned state variables and $N_M \geq 0$ macroeconomic variables can be written in the form above. Extending it to more than two spanned state variables is straightforward.

3.1 Time-Varying Volatility

The Gaussian model produces tractable affine pricing, but the volatility of commodity futures clearly varies over time (Trolle and Schwartz (2009)). Time varying volatility affects futures prices directly via the convexity term and might also affect price forecasts or expected returns. If these effects are present, in general they will be reflected in the reduced-form (pricing) factors because the model identifies the spanned state variables directly from futures prices in an agnostic way. Thus, any spanned effects of stochastic volatility on expected returns and price forecasts are compatible with the estimates and do not confound the findings.

Additional forecasting regressions also show that unspanned stochastic volatility does not explain the association between real activity and oil futures returns. Appendix A shows the results of forecasting regressions when I include measures of oil futures volatility. All three volatility measures are insignificant in forecasting oil futures returns and price changes, and more importantly they do not alter the forecasting power of GRO. The fact that time-varying volatility does not subsume the forecasting power of GRO is also consistent with the finding of Chiang, Hughen and Sagi (2015) that exposure to crude oil volatility has a risk premium attached to it in equities, but not in oil futures.

4 Model Estimates

Table 2 presents the parameters of the maximum likelihood estimate of the model. The coefficient of Δs_{t+1} on GRO_t is positive and statistically significant at the 1% level, echoing the forecasting regressions in Table 1. Note that a higher value of GRO_t also forecasts a fall in the cost of carry Δc_{t+1}, whereas Table 1 Panel B shows that higher GRO_t weakly forecasts a higher value of the slope factor. The state variables (s_t, c_t) are not the same as the level and slope factors in futures prices, and hence a VAR using the principal components plus the macroeconomic factors would not deliver the same results.

Similar to the PCs, the model-implied spanned factors (s_t, c_t) do a good job of summarizing the term structure of oil futures prices.[9] The model fitted values for f_t explain 99.97% of variation in observed log futures prices and the residuals (pricing errors) explain 0.03%. The root mean squared pricing error (RMSE) of the model is 54 basis points, in line with previous futures pricing models.

[9]This does not contradict the conclusions of e.g. Schwartz (1997) and Casassus and Collin-Dufresne (2005) that a three-factor model is necessary to summarize commodity futures prices. The three-factor models in those papers have two latent factors – spot price and convenience yield – and a spanned interest rate that is estimated separately. Interest rates are very slow moving compared to futures prices, so they contribute almost no explanatory power.

Table 2: Maximum likelihood (ML) estimate of the macro-finance model for Nymex crude oil futures using data from January 1986 to June 2014. s_t, c_t are the spot price and annualized cost of carry respectively. GRO_t is the monthly Chicago Fed National Activity Index. INV_t is the log of the private U.S. crude oil inventory as reported by the EIA. The coefficients are over a monthly horizon, and the time series are de-meaned. ML standard errors are in parentheses.

	$K_0^{\mathbb{P}}$	$K_1^{\mathbb{P}}$			
		s_t	c_t	GRO_t	INV_t
Δs_{t+1}	0.009	-0.007	0.060^{*}	0.025^{***}	0.045
	(0.006)	(0.008)	(0.031)	(0.008)	(0.076)
Δc_{t+1}	-0.007	0.017^{**}	-0.119^{***}	-0.015^{*}	-0.074
	(0.005)	(0.008)	(0.029)	(0.007)	(0.071)
ΔGRO_{t+1}	-0.002	-0.094^{**}	-0.022	-0.380^{***}	0.028
	(0.030)	(0.044)	(0.163)	(0.042)	(0.404)
ΔINV_{t+1}	0.002	0.001	0.029^{***}	-0.001	-0.089^{***}
	(0.002)	(0.002)	(0.008)	(0.002)	(0.020)

	$K_0^{\mathbb{Q}}$	$K_1^{\mathbb{Q}}$	
		s_t	c_t
Δs_{t+1}	-0.003	0.000	0.083^{***}
	(0.007)	(0.003)	(0.011)
Δc_{t+1}	-0.001	-0.004	-0.106^{***}
	(0.011)	(0.009)	(0.030)

	Shock Volatilities			
	s	c	GRO	INV
s	0.100			
c	-81%	0.056		
GRO	5%	-2%	0.527	
INV	-21%	25%	4%	0.026

4.1 Dynamics of the State Variables

The models of Schwartz (1997) and Schwartz and Smith (2000) impose the restriction that the spot price s_t is a random walk. Without that restriction and using data from 1990 to 2003, Casassus and Collin-Dufresne (2005) estimate that s_t is mean reverting with a half-life of two years, so the expected spot price of oil in ten years' time is effectively constant. The unrestricted estimate in Table 2 which adds ten years of subsequent data is more consistent with Schwartz (1997). The coefficient of Δs_{t+1} on s_t is -0.007 and is statistically not distinct from zero.[10]

The cost of carry reverts to a slightly negative mean with a half-life of five months. Shocks to the spot price and the cost of carry are strongly negatively correlated ($\rho = -81\%$), so a higher spot price is accompanied by a more downward sloping curve, but spot price shocks are essentially permanent while the cost of carry shock decays within a few years. As a result, about half of a typical move in the oil spot price disappears after two to three years, while the other half is expected to persist effectively forever.

Figure 3 Panel A plots the components of GRO that are spanned and unspanned by (s_t, c_t). We see that effectively all of the monthly and yearly variation in GRO appears in the unspanned component. Figure 3 Panel B plots the spanned and unspanned components of log oil inventories INV. Compared to GRO, much more of the monthly and yearly variation in INV is spanned by log futures prices. The spanned component of inventory loads exclusively and strongly on the cost of carry c_t.

[10]The estimates all use nominal futures prices. Inflation was slow moving over the period from 1986 to 2014 relative to movements in oil prices, so it has little effect on the dynamics of s_t; using futures prices deflated by the CPI or PPI does not change any of the estimates in the paper.

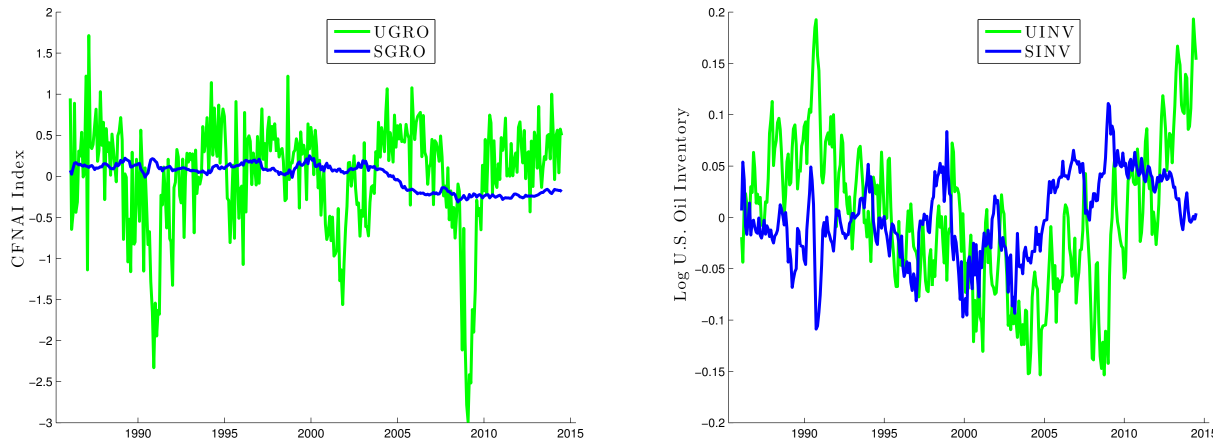

Figure 3: Panel A plots the components of the monthly Chicago Fed National Activity Index GRO that are spanned ($SGRO$) and unspanned ($UGRO$) by log oil futures prices. Panel B plots the components of monthly log U.S. oil inventories INV that are spanned ($SINV$) and unspanned ($UINV$) by log oil futures prices.

4.1.1 Oil Prices and Real Activity

A one percent shock to real activity forecasts a 2.5% higher spot price of oil and a 1.5% lower cost of carry. On net, the effects of real activity on oil prices are expected by the market to be persistent – higher real activity raises both the short run and the expected long run price of oil.

Conversely, a higher spot price of oil forecasts lower real activity. A higher cost of carry – higher expected prices in future – forecasts slightly higher real activity, but c_t naturally forecasts a higher spot price as well. The impulse response functions in Section 4.1.3 make clear that the net effect of c_t on GRO is negative. As a result, a shock to the spot price of oil that the market expects to persist has a more negative effect on growth than a shock that is expected to be transitory.

In sum, there is a negative feedback relationship between the spot price of oil and real economic activity. A positive shock forecasts persistent higher oil prices, while a positive oil price shock forecasts lower real activity, and the effect is stronger for oil price shocks that the market expects to persist.

4.1.2 Oil Prices and Inventories

Shocks to log inventories are negatively correlated with the spot price and positively correlated with the cost of carry. Both of these observations are consistent with the Theory of Storage – higher inventories signal that the market is moving up the supply-of-storage curve. The correlation between shocks to inventory and the cost of carry (27%) is relatively modest; in the frictionless storage model of Working (1949) and others, INV_t and c_t are collinear. A higher cost of carry strongly predicts higher inventories the next month. The forecasting power of c_t for inventories suggests adjustment costs in physical storage: the futures curve adjusts to new information first and inventories respond with a lag.

Looking down the last column of the transition matrix, unspanned crude oil inventory does not forecast any of the other variables. In particular, periods of higher inventory do not have much effect on the forecast of either the spot price or the cost of carry. This finding is consistent with the fundamental drivers of oil inventory such as precautionary storage and expected physical supply and demand being fully spanned by oil futures prices.

4.1.3 Impulse Response Functions

Figure 4 plots the impulse response functions (IRFs) to shocks to oil prices and economic activity. The ordering of the variables for the impulse response functions is (GRO, s_t, c_t, INV). GRO is first because innovations in the unspanned component, which dominates the variation in GRO, can be thought of as exogenous to contemporaneous oil prices and inventories. We analyze s_t and c_t simultaneously so their relative ordering is not important. Finally, it

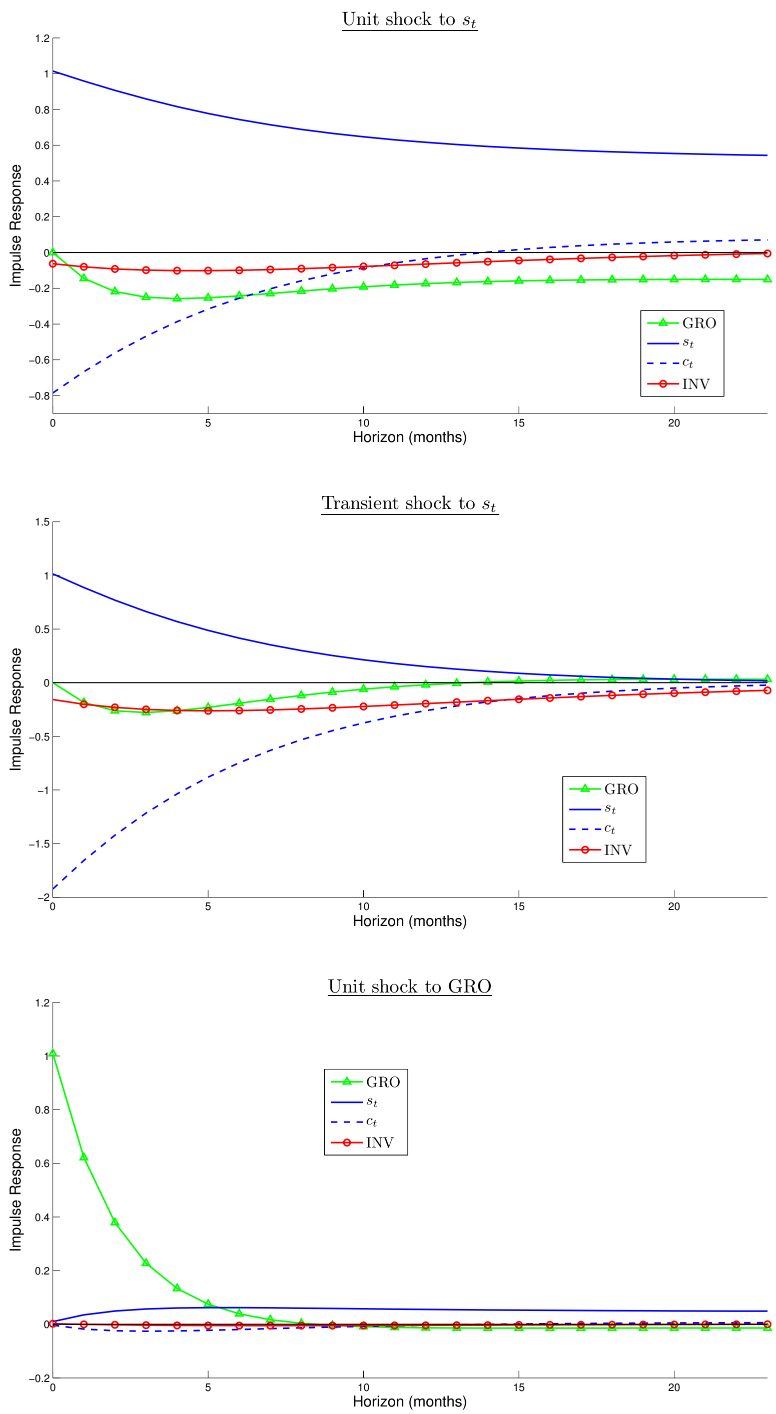

Figure 4: Panel A shows the impulse response functions (IRFs) of the four state variables to a unit shock to the log spot price of oil s_t. Panel B shows the IRFs for a transient shock for which the spot price of oil fully reverts to the baseline. Panel C shows the IRFs for a unit shock to economic growth, GRO. The order of the variables is (GRO, s_t, c_t, INV).

is intuitive and also supported by the estimates and regressions that the oil futures curve adjusts to new information faster than physical inventory does.

Panel A plots the response to a unit shock to the log spot price, which is correlated with a negative shock to the cost of carry and a more downward-sloping curve. A unit shock to s_t means a doubling of the spot price of oil. About half of the increase decays within two years, while the other half is effectively permanent, and forecasts an economic activity index that is 0.2% lower effectively forever. This effect is material: the index averaged -1.66% in 2009 during the depths of the financial crisis, while it averaged 0.02% in 2006. The higher spot price and lower cost of carry also produce a fall in inventories.

Panel B plots the response to a joint shock to s_t and c_t such that the spot price is expected to fully revert to the pre-shock baseline. The response of economic activity is transient as well, and in fact *GRO* recovers to the baseline faster than s_t does. Comparing to Panel A, which only differs in the size of the shock to c_t, makes clear that the net effect of c_t on expected growth is negative. Note that the fact that the forecast of the long-run spot price is unchanged in Panel B does not mean that long maturity futures prices will be unchanged – the two are equivalent only in the case that oil risk premiums are non time varying. Thus, a VAR that includes a long-maturity futures price or spread will not in general recover the correct dynamics of the state variables.

Panel C plots the response to a shock to economic activity. The index mean reverts rapidly and the shock decays back to the baseline within a year. However, a transient shock to *GRO* produces a near permanently higher spot price of oil – perhaps because oil is a nonrenewable resource. The magnitude of the effect is large: a one-period shock to economic activity of one percent produces a spot price of oil that is 5.1% higher than the baseline, ten years later.

Table 3: Maximum likelihood (ML) estimates of the parameters governing risk premiums in the macro-finance model for U.S. crude oil futures. s_t, c_t are the spot price and annualized cost of carry respectively. GRO and INV are the Chicago Fed National Activity Index and log U.S. crude oil inventory respectively. The coefficients are standardized to reflect a one standard deviation change in each variable over a monthly horizon, and the time series are de-meaned. ML standard errors are in parentheses.

$$\left[\begin{array}{c} \Lambda^s \\ \Lambda^c \end{array}\right]_t = \Lambda_0 + \Lambda_1 \left[\begin{array}{ccc} s_t & c_t & M_t \end{array}\right]'$$

	Λ_0	Λ_1			
		s_t	c_t	GRO_t	INV_t
Λ^s	0.011	-0.001	-0.001	0.013***	0.001
	(0.013)	(0.001)	(0.002)	(0.004)	(0.002)
Λ^c	-0.006	0.003	-0.001	-0.008*	-0.002
	(0.016)	(0.002)	(0.003)	(0.004)	(0.002)

4.2 Risk Premiums

Table 3 displays the estimates of the parameters governing time-varying risk premiums. The unconditional spot risk premium is positive, while the unconditional cost-of-carry risk premium is negative.[11] Two entries in the time-varying loadings of risk premiums Λ_1 are statistically significant: higher real activity GRO is associated with a higher spot risk premium and a lower cost-of-carry risk premium in oil. As in the forecasting regressions, this finding is inconsistent with the risk spanning assumption that is implicit in previous pricing models.

The effect of economic activity on the estimated spot risk premium in oil futures is material. Figure 5 plots the implied spot premiums for the macro-finance model and the two-

[11] Szymanowska et al. (2013) decompose futures returns into a spot premium and a term premium based on different trading and rolling strategies. The Internet Appendix describes the relation between their decomposition and the risk premiums in the model. Briefly, their spot premium equals the risk premium attached to the spot price plus a small convexity term, while their term premium equals the risk premium attached to the cost of carry minus the conditional expected cost of carry.

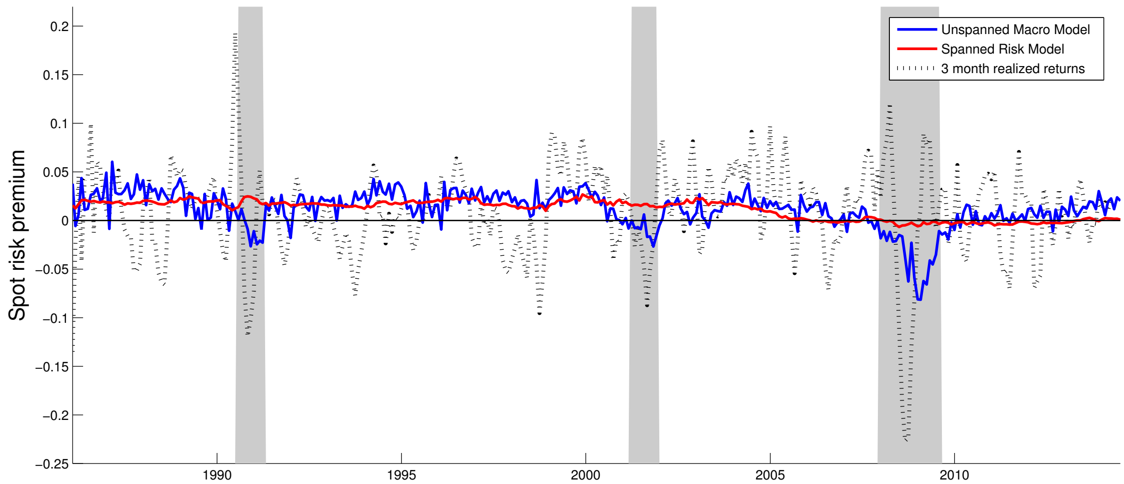

Figure 5: The figure compares the spot risk premium in oil futures from the model with unspanned macro factors versus the nested model that enforces spanning. Also plotted is the average return across all active oil futures over the following three months. NBER recessions are shaded in grey.

factor nested model that enforces spanning, as well as average realized returns for oil futures in the sample over the following three months. The model predictions differ most noticeably during the recessions of 1990-1991, 2001-2002 and 2008-2009: slumps in real activity forecast falling oil prices. The unspanned procyclical component dominates the variation in the spot risk premium; the standard deviation of changes in Λ_t^s in the unspanned macro model is 1.47% per month compared to 0.17% per month in the spanned-risk model, an increase of nearly tenfold.

The macro-finance model does significantly better at explaining both returns to the level portfolio, as Figure 5 suggests, and the full panel of futures returns. Taking the panel of log futures returns and comparing them with their predicted values:

$$r_{t+1}^j = f_{t+1}^{j-1} - f_t^j, \ j = 2, ..., J, \ t = 0, ..., T-1$$

$$\lambda_{t+1}^{j} = B_j \Lambda_t$$

$$resid_{t+1}^{j} = r_{t+1}^{j} - \lambda_{t+1}^{j}$$

the adjusted R^2 of the macro-finance model is 4.3% compared to 1.3% for the nested spanned-risk model. Thus, after adjusting for the added degrees of freedom, adding the unspanned macro factors more than triples the fractiaon of oil futures returns that the model is able to explain.

The forecasting power for returns is attached to the unspanned component of real activity because it is not reflected in the futures curve at the time. Per the estimates in Table 2, shocks to real activity and the cost of carry are negatively correlated. A fall in GRO is correlated with a fall in the spot price s_t but a slight rise in c_t and the slope of the futures curve. In other words, when real activity falls, long maturity oil futures fail to forecast the subsequent fall in the spot price.

4.3 Oil Futures as a Consumption Hedge

A potential motivation for the procyclical expected returns in oil futures is as follows. Suppose a representative agent consumes oil O_t and a general consumption good C_t, with

$$V_t = E\left[\sum_{t=0}^{\infty} \beta^t \frac{A_t^{1-\gamma}}{1-\gamma}\right]$$

$$A(C_t, O_t) = \left[C_t^{1-\rho} + \omega O_t^{1-\rho}\right]^{\frac{1}{1-\rho}}$$

I assume that $\gamma > \rho > 1$, that is, oil and the general consumption good are complements and investors' risk aversion over total consumption is stronger than the elasticity between the two goods. Normalize the price of the consumption good to 1 and denote by P_t^O the price of a barrel of oil. The intratemporal equilibrium is

$$P_t^O = \omega \left(\frac{C_t}{O_t} \right)^{\rho}$$

and the stochastic discount factor is[12]

$$\mathcal{M}_{t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \left(\frac{1 + \omega O_{t+1}^{1-\rho} / C_{t+1}^{1-\rho}}{1 + \omega O_t^{1-\rho} / C_t^{1-\rho}} \right)^{\frac{\rho-\gamma}{1-\rho}}$$

Log linearizing,

$$m_t \approx K - \gamma - \omega(\gamma - \rho))\Delta c_{t+1} - \omega(\gamma - \rho)\Delta o_{t+1}$$

where K is a constant. The expected log return on any asset is

$$E\left[r_{t+1}\right] = r_t^f + (\gamma - \omega(\gamma - \rho))cov(r_{t+1}, \Delta c_{t+1}) + \omega(\gamma - \rho)cov(r_{t+1}, \Delta o_{t+1})$$

where r_t^f is the log risk free rate. Thus, assets pay a consumption risk premium for exposure to c_t shocks and an oil risk premium for exposure to o_t shocks. Note that this prediction does not assume anything about the dynamics of c_t or o_t – consumption of oil and the general good may be exogenous or endogenous.

A one period oil future has a loading of 1 on the spot price of oil and hence a loading of $-\rho$ on o_t. Thus, consumers pay a hedge premium to be long oil futures because it insures them against shocks to the flow of oil. If risk aversion γ is time-varying and higher in recessions, as is well established across asset markets (Cochrane, 2011), then expected returns to oil

[12] This derivation follows, e.g., Yogo (2006).

futures will be procyclical because the hedge premium is higher in recessions.[13]

Both regressions and the model estimate indicate that real activity is not reflected in contemporaneous oil futures prices.[14] This corresponds to real activity having offsetting effects on oil risk premiums and the spot price forecast. We observe in the data that higher real activity is followed by higher spot prices of oil but that oil futures do not slope upward to reflect that forecast. A positive growth shock lowers the hedge premium, which *lowers* oil futures prices. At the same time, it forecasts higher demand and a higher spot price of oil, which *raises* oil futures prices, consistent with the net effect being unspanned in the contemporaneous futures curve.

4.4 Hedging Pressure: Producers versus Consumers

The hypothesis outlined above suggests that real activity drives oil risk premia via demand for hedging oil consumption. This explanation is in contrast with the "hedging pressure" theory that originated with Keynes (1930), which says that risk premia in commodity futures markets are due to hedging demand from commodity producers.

The track record of hedging pressure as an explanation of risk premia in commodity futures markets is poor. Most studies find that hedging pressure does not forecast returns or prices in most futures markets (e.g.Bessembinder (1992); Gorton, Hayashi and Rouwenhorst (2013)). I reach the same conclusion. Table 4 shows the estimated parameters that govern oil risk premia in the model using $M_t = [GRO_t, HP_t]$ where HP is the hedging pressure in oil futures computed from the CFTC's Commitment of Traders Report.[15] We see that

[13]In contemporaneous research Ready (2015) and Baker, Steven D. and Routledge (2015) solve models that generate an endogenously time-varying hedge premium to oil futures.

[14] The existence of macro variables that are unspanned by bond yields is an active question in the term structure literature (Duffee, 2011; Chernov and Mueller, 2012; Joslin, Priebsch and Singleton, 2014).

[15]Hedging pressure equals the net position among commercial participants divided by the total open

Table 4: Maximum likelihood (ML) estimate of the transition matrix for the model in which $M_t = [GRO, HP]$ where GRO and HP are the monthly Chicago Fed National Activity Index and the hedging pressure calculated from the CFTC's Commitment of Traders reports. The coefficients are over a monthly horizon, and the time series are de-meaned. ML standard errors are in parentheses.

	$K_0^{\mathbb{P}}$	$K_1^{\mathbb{P}}$			
		s_t	c_t	GRO_t	HP_t
Δs_{t+1}	0.007	-0.012	0.075***	0.023***	-0.083
	(0.006)	(0.009)	(0.028)	(0.008)	(0.069)
Δc_{t+1}	-0.006	0.022**	−0.139***	-0.013	0.088
	(0.005)	(0.009)	(0.026)	(0.008)	(0.065)
ΔGRO_{t+1}	-0.018	−0.148***	0.040	−0.395***	−0.776**
	(0.030)	(0.050)	(0.146)	(0.043)	(0.364)
ΔHP_{t+1}	-0.010	−0.012**	−0.037**	−0.011**	−0.257***
	(0.003)	(0.005)	(0.016)	(0.005)	(0.039)

hedging pressure does not drive oil risk premia, nor does it subsume the strong relationship of real activity with the oil risk premium. On the other hand, we see that higher current oil prices s_t or expected future oil prices via c_t both forecast less hedging pressure i.e. fewer commercial participants going long. This conclusion based on the time series of oil futures returns across the full term structure of oil futures is consistent with Gorton, Hayashi and Rouwenhorst (2013)'s conclusion using portfolio sorts across the cross-section of commodity futures.

4.5 Positive vs Negative Growth Regimes

Looking at Figure 5, the effect of real activity on oil prices appears to be concentrated in economic downturns. To investigate this possibility I split GRO into two components:

interest, as of the most recent Commitment of Traders report.

GRO^+ equals GRO in months when the lagged three month average of GRO is positive and equals zero otherwise, and GRO^- equals GRO in months when the lagged three month average of GRO is negative and equals zero otherwise.[16] This split lets the coefficients differ when real activity is in a positive-growth regime versus a negative-growth regime.

Table 5 presents the estimated feedback matrix $K_1^{\mathbb{P}}$. Contrary to our impression from Figure 5, the effect of real activity on the oil prices seems to be symmetrical in good times versus bad – the point estimates of the coefficients of Δs_{t+1} on GRO^- and GRO^+ are precisely equal. On the other hand, the estimate suggests an asymmetry on the 'supply' side. The forecast of real activity is impaired by oil price shocks in positive growth regimes: the coefficient of ΔGRO^+_{t+1} on s_t is -0.082 and is statistically significant at the 1% level. By comparison the coefficient of ΔGRO^-_{t+1} on s_t i.e. in negative growth regimes is less than half as large and is not statistically significant. Thus, while the effect of shocks to real activity on the oil price forecast is relatively symmetric in good times and bad, the negative effect of oil price shocks on the forecast of real activity is stronger in times when growth is positive.

4.6 Subcomponents of Real Activity

The Chicago Fed divides the 85 constituent time series of the CFNAI into four categories which form subcomponents of the real activity index. The subcomponents are Industrial Production and Income (PI); Employment and Hours (EUH); Personal Consumption and Housing (CH); and Sales Orders and Inventories (SOI). When I estimate the model with the four subcomponents $M_t = [PI_t, EH_t, PCH_t, SI_t]$ I find that employment EUH and sales and inventories SOI do not interact significantly with oil prices, but that industrial production PI and personal consumption PCH do.

[16] As the unspanned components of the macroeconomic series do not enter into contemporaneous prices, they need not be Gaussian.

Table 5: Maximum likelihood (ML) estimate of the transition matrix for the model in which GRO^+ and GRO^- are the monthly Chicago Fed National Activity Index in months when the lagged three month average of GRO is positive and negative respectively. The coefficients are over a monthly horizon, and the time series are de-meaned. ML standard errors are in parentheses.

	$K_0^{\mathbb{P}}$	$K_1^{\mathbb{P}}$			
		s_t	c_t	GRO_t^+	GRO_t^-
Δs_{t+1}	0.009	-0.006	0.069**	0.025	0.025**
	(0.007)	(0.008)	(0.027)	(0.016)	(0.010)
Δc_{t+1}	-0.009	0.016**	−0.133***	-0.009	−0.018*
	(0.006)	(0.008)	(0.026)	(0.015)	(0.010)
ΔGRO_{t+1}^+	0.147	−0.082***	-0.048	−0.789***	0.076**
	(0.022)	(0.028)	(0.092)	(0.052)	(0.034)
ΔGRO_{t+1}^-	-0.052	-0.038	0.052	0.036	−0.281***
	(0.025)	(0.031)	(0.102)	(0.058)	(0.038)

Table 6 presents the results of estimating the model using those two subcomponents of the real activity index, $M_t = [PI_t, PCH_t]$. We see that the "demand" channel in which shocks to real activity affect the oil price forecast appears to be driven through industrial production. The coefficient of Δs_{t+1} on PI_t is positive (0.141) and statistically significant while the coefficient of Δs_{t+1} on PCH_t is much smaller and is not statistically significant. Also, shocks to PI are contemporaneously correlated with a higher spot price of oil s_t. By contrast the "supply" channel in which shocks to the oil price affect real activity appears to be driven by personal consumption. The coefficient of ΔPCH_{t+1} on s_t is negative and statistically significant, and the coefficient of ΔPCH_{t+1} on c_t (expected future oil prices) is also negative, while the coefficient of ΔPI_{t+1} on s_t is much smaller and is not statistically significant.

This decomposition suggests another asymmetry in the dynamic between oil prices and

Table 6: Maximum likelihood (ML) estimate of the dynamics for the model in which $M_t = [PI_t, PCH_t]$ are the subcomponents of the Chicago Fed National Activity Index that measure industrial production and income (PI) and personal consumption and housing (PCH) respectively. The coefficients are over a monthly horizon, and the time series are de-meaned. ML standard errors are in parentheses.

	$K_0^{\mathbb{P}}$	$K_1^{\mathbb{P}}$			
		s_t	c_t	PI_t	PCH_t
Δs_{t+1}	0.009	-0.007	0.069**	0.141**	0.023
	(0.006)	(0.009)	(0.029)	(0.053)	(0.039)
Δc_{t+1}	-0.008	0.015	−0.135***	-0.085	-0.021
	(0.005)	(0.009)	(0.027)	(0.050)	(0.037)
ΔPI_{t+1}	0.001	-0.001	0.024	−0.728***	0.070*
	(0.006)	(0.009)	(0.028)	(0.053)	(0.039)
ΔPCH_{t+1}	-0.001	−0.013**	-0.016	-0.047	−0.090***
	(0.003)	(0.006)	(0.017)	(0.032)	(0.023)

	Shock Volatilities			
	s	c	PI	PCH
s	0.101			
c	-81%	0.056		
PI	15%	-10%	0.098	
PCH	3%	-2%	25%	0.057

the real economy. On one hand, shocks to industrial productivity are associated with higher oil prices both contemporaneously and with a lag, while shocks to consumer spending have little or no effect. On the other hand, oil shocks affect real activity through falling consumption spending which reacts with a lag to a higher spot price.

5 Real Options

Firms' capacity to adjust their investment or production ex post make up a substantial part of firm value, and evaluating and exercising those adjustments is a primary role of firm management (Pindyck (1988); Berk, Green and Naik (2004); Carlson, Fisher and Giammarino (2004)). Previous studies such as Brennan and Schwartz (1985) and Casassus and Collin-Dufresne (2005) have explored what commodity derivatives markets can tell us about the value of real options. These studies, which use spanned-risk models, make the implicit assumption that all relevant factors are spanned by commodity futures. In Trolle and Schwartz (2009) and Chiang, Hughen and Sagi (2015), a latent volatility factor is unspanned in oil futures but spanned by oil options.

By contrast, if a macro factor M_t is unspanned in the sense of this paper then it does not affect the price of *any* derivatives on the underlying commodity. Unspanned factors of this type are still relevant to real options, however, if the option's payoff depends on M_t. For example, an oil well is often modeled as the right to pump oil out of the ground at a fixed cost per barrel, equivalent to a purely financial option. But for a real oil well the costs of extraction are uncertain. Moel and Tufano (2002) find that for gold mines, changing extraction costs over time are a significant predictor of mine openings and closings after controlling for factors estimated from both futures and options on gold. More generally, commodity prices are only one element of the firm's decision process. For example, in an airline's decision to purchase more fuel efficient planes, the cost savings will vary with oil prices while revenues will vary with aggregate economic activity. Pindyck (1993) makes this argument and points out that relevant macroeconomic risk factors will affect real option valuation[17].

[17] "... *this effect [of uncertainty on option value and exercise] is magnified when fluctuations in construction*

To examine the effects of unspanned macroeconomic risks on real options valuation, I model an oil well as a ten year strip of European options on an oil field that produces 1000 barrels of oil per month when open. The oil is extracted at lifting cost L_t and sold at the spot price S_t each month that it is open. Thus, it produces in any month when $S_t > L_t$.

I simulate data for the state variables (s, c, GRO) and the log lifting cost l_t which is a function of all three. That is, I assume that l_t has spanned and unspanned components plus an idiosyncratic noise term. Appendix B describes the setting in detail. I then estimate different model specifications on the same simulated data, and compute the fair price of the oil well over a range of current lifting costs L_0.

Figure 6 plots the value of oil wells with different current lifting costs, using different models estimated on the same data. The lower two lines represent option values for spanned-risk models in which all relevant risks are assumed to be spanned by oil futures. This means l_t must be a linear combination of s_t and c_t plus an error term (Joslin, Le and Singleton (2013)). Whether the error term is modelled as an i.i.d. or AR(1) process is essentially irrelevant to option value.

The upper two lines represent option values from models that incorporate unspanned macro risk. We see that the spanned-risk models miss a large component of option value. To emphasize, the monthly volatility of l_t in the spanned-risk model is the same as it is in the unspanned-risk models: the difference is that l_t's dependence on GRO adds persistent time variation in lifting costs that covaries with the spot price and cost of carry. This addition has a large effect on option valuation: Adding the unpriced ($\lambda = 0$) unspanned macro risk raises the real option value by 35% for an 'in the money' well with current lifting cost =

costs are correlated with the economy, or, in the context of the Capital Asset Pricing Model, when the 'beta' of cost is high... [A] higher beta raises the discount rate applied to expected future costs, which raises the value of the investment opportunity as well as the benefit from waiting rather than investing now."

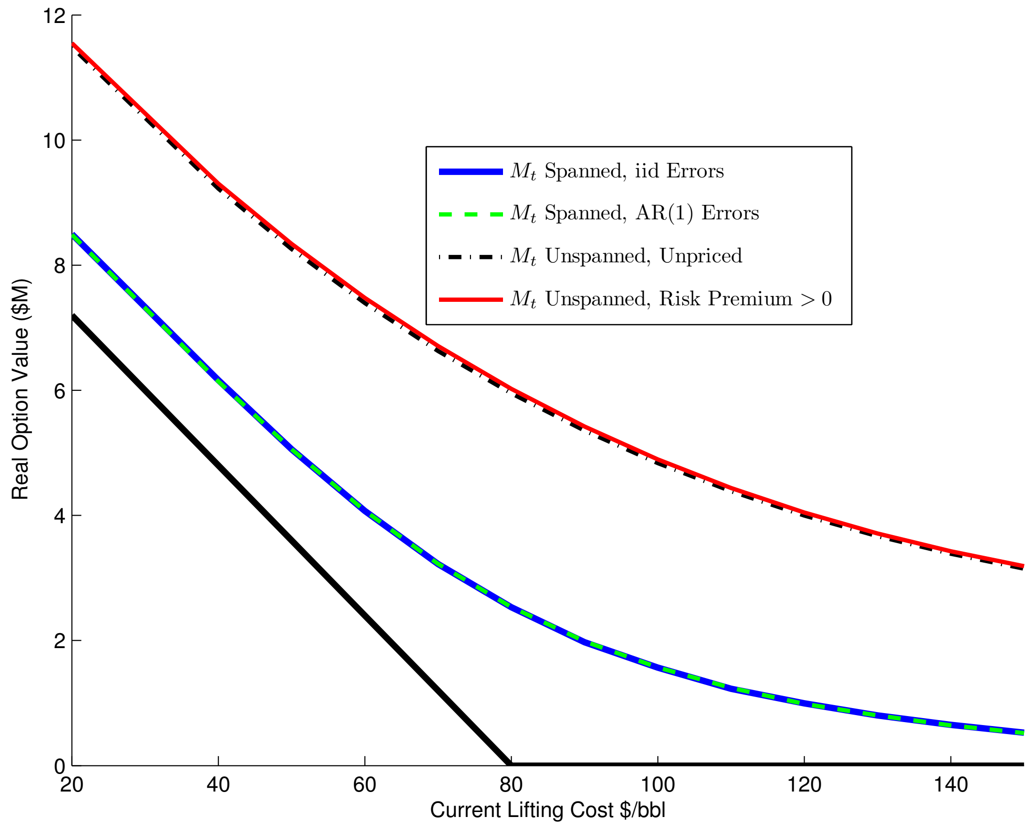

Figure 6: Real options valuation with unspanned macro risk. An oil well is modelled as a strip of European options that are exercised when the stochastic log extraction cost l_t is less than the log spot price s_t. l_t covaries with s_t and the unspanned macro risk GRO_t. The current spot price of oil is \$80, and the x-axis indexes the current lifting cost L_0 of different oil wells.

\$20 per barrel and 405% for an 'out of the money' well with current lifting cost of \$150 per barrel.

The risk premium effect is that the option value is higher when GRO, and hence L_t, carries a positive risk premium ($\lambda > 0$). This effect on valuation is present but small, increasing the well's value by only 0.99% for the 'in the money' well with $L_0 = 20$ and by 1.27% for the 'out of the money' well with $L_0 = 150$.

6 Conclusion

This paper investigates the interaction of futures markets with the real economy. I develop a macro-finance futures pricing model that admits unspanned macroeconomic variables. The model generalizes and extends benchmark futures pricing models, and connects them with macroeconomic studies that use vector autoregressions (VARs).

Both forecasting regressions and model estimates reveal a novel empirical fact: real activity forecasts oil prices and the effect is unspanned in contemporaneous oil futures. The estimated oil risk premium in the model is strongly procyclical and nine times more volatile than the nested model that assumes complete spanning, which suggests that spanned-risk models may miss the majority of variation in futures risk premia. This pattern in the oil risk premium is consistent with countercyclical hedging demand by oil consumers.

By construction, unspanned macro factors do not affect the price of any commodity derivatives. However, when the payoff of a real option depends on macroeconomic factors beyond the commodity price then unspanned macro factors can have a large effect on real option value and exercise. In a calibrated example I show that both the dynamics and the risk premiums of unspanned macro risks raise the values of a hypothetical real option significantly relative to a benchmark spanned-risk model.

The model estimates reveal rich dynamics of oil prices with the economy. Higher oil prices forecast lower real activity, especially when the price increase is forecast by the market to be persistent, and especially when the economy is in an expansion period. Higher real activity forecasts a higher oil price, and although real activity shocks are transient the resulting higher oil price is highly persistent, which may be related to the fact that oil is a nonrenewable resource. The channel from real activity to oil prices flows through industrial production, while the channel from oil prices to real activity flows through consumer spending.

To my knowledge these are the first estimates of a macro-finance style model for commodity futures which incorporates explicit macroeconomic factors. The estimates presented in this paper are all maximally flexible, meaning that the model imposes no overidentifying restrictions on the dynamics or risk prices of the state variables. Thus, the estimates are best seen as summarizing the data in a consistent and parsimonious way. A natural direction for further research will be to impose well-motivated restrictions on the dynamics of the state variables as in the VAR literature (Barsky and Kilian, 2004; Kilian, 2009) and on risk prices as in the futures pricing literature (Schwartz, 1997; Casassus, Liu and Tang, 2013), and further explore their interactions.

References

Alquist, R and Kilian, L, What do we learn from the price of crude oil futures? Journal of Applied Econometrics, 25 2010:4, pp. 539–573.

Ang, A and Piazzesi, M, A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables. Journal of Monetary Economics, 50 2003:4, pp. 745–787 ⟨URL: http://www.sciencedirect.com/science/article/pii/S0304393203000321⟩.

Aruoba, S. Boragan, Diebold, Francis X. and Scotti, Chiara, Real-Time Measurement of Business Conditions. Journal of Business & Economic Statistics, 27 2009:4, pp. 417–427 ⟨URL: http://amstat.tandfonline.com/doi/abs/10.1198/jbes.2009.07205⟩, ISSN 0735–0015.

Baker, Steven D. and Routledge, Bryan R., The Price of Oil Risk. 2015 ⟨URL: https://www.bus.miami.edu/_assets/files/faculty-and-research/conferences-and-seminars/finance-seminars/RoutledgePaper.pdf⟩.

Barsky, R and Kilian, L, Oil and the Macroeconomy since the 1970s. Journal of Economic Perspectives, 18 (4) 2004, pp. 115–134 ⟨URL: http://www.nber.org/papers/w10855⟩.

Berk, Jonathan B, Green, Richard C and Naik, Vasant, Valuation and return dynamics of new ventures. Review of Financial Studies, 17 2004:1, pp. 1–35 ⟨URL: http://rfs.oxfordjournals.org/content/17/1/1.short⟩.

Bernanke, B S et al., Systematic monetary policy and the effects of oil price shocks. Brookings papers on economic activity, 1997 1997:1, pp. 91–157.

Bessembinder, H., Systematic Risk, Hedging Pressure, and Risk Premiums in Futures Markets. Review of Financial Studies, 5 October 1992:4, pp. 637–667 ⟨URL: http://rfs.oxfordjournals.org/content/5/4/637.short⟩, ISSN 0893–9454.

Bessembinder, Hendrik and Chan, Kalok, Time-varying risk premia and forecastable returns in futures markets. Journal of Financial Economics, 32 1992:2, pp. 169–193 ⟨URL: http://www.sciencedirect.com/science/article/pii/0304405X9290017R⟩.

Brennan, M J and Schwartz, E S, Evaluating natural resource investments. Journal of Business, 1985, pp. 135–157 ⟨URL: http://www.jstor.org/stable/10.2307/2352967⟩.

Carlson, Murray, Fisher, Adlai and Giammarino, Ron, Corporate Investment and Asset Price Dynamics: Implications for the Cross-section of Returns. The Journal of Finance, 59 2004:6, pp. 2577–2603 ⟨URL: http://onlinelibrary.wiley.com/doi/10.1111/j.1540-6261.2004.00709.x/full⟩.

Casassus, J and Collin-Dufresne, P, Stochastic Convenience Yield implied from Commodity Futures and Interest Rates. The Journal of Finance, 60 (5) 2005, pp. 2283–2331 ⟨URL: http://onlinelibrary.wiley.com/doi/10.1111/j.1540-6261.2005.00799.x/abstract⟩.

Casassus, Jaime, Liu, Peng and Tang, Ke, Economic linkages, relative scarcity, and commodity futures returns. Review of Financial Studies, 26 (5) 2013, pp. 1324–1362 ⟨URL: http://rfs.oxfordjournals.org/content/26/5/1324⟩.

Chen, NF, Roll, R and Ross, SA, Economic forces and the stock market. Journal of Business, 1986 ⟨URL: http://www.jstor.org/stable/2352710⟩.

Chernov, Mikhail and Mueller, Philippe, The term structure of inflation expectations. Journal of Financial Economics, 106 November 2012:2, pp. 367–394 ⟨URL: http://www.sciencedirect.com/science/article/pii/S0304405X12001195⟩, ISSN 0304405X.

Chiang, I-Hsuan Ethan, Hughen, W. Keener and Sagi, Jacob S., Estimating Oil Risk Factors Using Information from Equity and Derivatives Markets. The Journal of Finance, 70 April 2015:2, pp. 769–804 ⟨URL: http://doi.wiley.com/10.1111/jofi.12222⟩, ISSN 00221082.

Cochrane, John H, Discount rates. The Journal of Finance, 66 2011:4, pp. 1047–1108 ⟨URL: http://onlinelibrary.wiley.com/doi/10.1111/j.1540-6261.2011.01671.x/full⟩.

Collin-Dufresne, Pierre, Goldstein, Robert S. and Jones, Christopher S., Identification of Maximal Affine Term Structure Models. The Journal of Finance, 63 April 2008:2, pp. 743–795 ⟨URL: http://doi.wiley.com/10.1111/j.1540-6261.2008.01331.x⟩, ISSN 0022–1082.

Cooper, I. and Priestley, R., Time-Varying Risk Premiums and the Output Gap. Review of Financial Studies, 22 October 2008:7, pp. 2801–2833 ⟨URL: http://rfs.oxfordjournals.org/content/22/7/2801.short⟩, ISSN 0893–9454.

Dai, Qiang and Singleton, Kenneth J, Specification analysis of affine term structure models. The Journal of Finance, 55 2000:5, pp. 1943–1978 ⟨URL: http://onlinelibrary.wiley.com/doi/10.1111/0022-1082.00278/abstract⟩.

Diebold, F X, Rudebusch, G D and Aruoba, S, The macroeconomy and the yield curve: a dynamic latent factor approach. Journal of econometrics, 131 2006:1, pp. 309–338 ⟨URL: http://www.sciencedirect.com/science/article/pii/S030440760500014X⟩.

Duffee, G R, Term premia and interest rate forecasts in affine models. The Journal of Finance, 57 2002:1, pp. 405–443 ⟨URL: http://onlinelibrary.wiley.com/doi/10.1111/1540-6261.00426/abstract⟩.

Duffee, G R, Information in (and not in) the term structure. Review of Financial Studies, 24 2011:9, pp. 2895–2934 ⟨URL: http://rfs.oxfordjournals.org/content/24/9/2895.short⟩.

Duffie, Darrell and Kan, Rui, A yield-factor model of interest rates. Mathematical finance, 6 1996:4, pp. 379–406 ⟨URL: http://onlinelibrary.wiley.com/doi/10.1111/j.1467-9965.1996.tb00123.x/full⟩.

Fama, E F and French, K R, Commodity futures prices: Some evidence on forecast power, premiums, and the theory of storage. Journal of Business, 1987, pp. 55–73.

Ferson, WE, Sarkissian, S and Simin, TT, Spurious regressions in financial economics? The Journal of Finance, 2003 ⟨URL: http://onlinelibrary.wiley.com/doi/10.1111/1540-6261.00571/full⟩.

Gibson, R and Schwartz, E S, Stochastic convenience yield and the pricing of oil contingent claims. Journal of Finance, 1990, pp. 959–976 ⟨URL: http://www.jstor.org/stable/10.2307/2328801⟩.

Gorton, Gary B, Hayashi, Fumio and Rouwenhorst, K Geert, The fundamentals of commodity futures returns. Review of Finance, 17 2013:1, pp. 35–105 ⟨URL: http://rof.oxfordjournals.org/content/17/1/35.short⟩.

Hamilton, J D, Oil and the macroeconomy since World War {II}. The Journal of Political Economy, 1983, pp. 228–248.

Hamilton, J D, What is an oil shock? Journal of Econometrics, 113 2003:2, pp. 363–398 ⟨URL: http://www.sciencedirect.com/science/article/pii/S0304407602002075⟩.

Hamilton, J D and Wu, J C, Risk Premia in Crude Oil Futures Prices. Journal of International Money and Finance, 42 2014, pp. 9–37.

Hamilton, James D. and Wu, Jing Cynthia, Effects of Index-Fund Investing on Commodity Futures Prices. International Economic Review, 56 February 2015:1, pp. 187–205 ⟨URL: http://doi.wiley.com/10.1111/iere.12099⟩, ISSN 00206598.

Hong, H and Yogo, M, What does futures market interest tell us about the macroeconomy and asset prices? Journal of Financial Economics, 2012 ⟨URL: http://www.sciencedirect.com/science/article/pii/S0304405X12000657⟩.

Joslin, S, Le, A and Singleton, K J, Why Gaussian Macro-Finance Term Structure Models Are (Nearly) Unconstrained Factor-VARs. Journal of Financial Economics, 109 (3) 2013, pp. 604–622 ⟨URL: http://www-bcf.usc.edu/~sjoslin/papers/JLS.pdf⟩.

Joslin, S, Priebsch, M and Singleton, K J, Risk Premiums in Dynamic Term Structure Models with Unspanned Macro Risks. Journal of Finance, 69 2014, pp. 1197–1233 ⟨URL: http://onlinelibrary.wiley.com/doi/10.1111/jofi.12131/abstract⟩.

Keynes, JM, A Treatise on Money. 1930 ⟨URL: https://scholar.google.com/scholar?hl=en\&q=keynes+treatise+on+money\&btnG=\&as_sdt=1\%2C45\&as_sdtp=\#1⟩.

Kilian, L, Not All Oil Price Shocks Are Alike: Disentangling Demand and Supply Shocks in the Crude Oil Market. American Economic Review, 99 2009:3, pp. 1053–1069.

Kilian, Lutz and Vega, Clara, Do energy prices respond to US macroeconomic news? A test of the hypothesis of predetermined energy prices. Review of Economics and Statistics, 93 2011:2, pp. 660–671 ⟨URL: http://www.federalreserve.gov/Pubs/ifdp/2008/957/ifdp957.pdf⟩.

Lamont, OA, Economic tracking portfolios. Journal of Econometrics, 2001 ⟨URL: http://www.sciencedirect.com/science/article/pii/S0304407601000744⟩.

Ludvigson, S C and Ng, S, Macro factors in bond risk premia. Review of Financial Studies, 22 2009:12, pp. 5027–5067.

Moel, Alberto and Tufano, Peter, When are real options exercised? An empirical study of mine closings. Review of Financial Studies, 15 2002:1, pp. 35–64 ⟨URL: http://rfs.oxfordjournals.org/content/15/1/35.short⟩.

Pindyck, Robert S, Irreversible Investment, Capacity Choice, and the Value of the Firm. American Economic Review, 78 1988:5, pp. 969–985 ⟨URL: http://ideas.repec.org/a/aea/aecrev/v78y1988i5p969-85.html⟩.

Pindyck, Robert S, Investments of uncertain cost. Journal of financial Economics, 34 1993:1, pp. 53–76 ⟨URL: http://www.sciencedirect.com/science/article/pii/0304405X9390040I⟩.

Ready, Robert C., Oil Consumption, Economic Growth, and Oil Futures: A Fundamental Alternative to Financialization. 2015 ⟨URL: http://rready.simon.rochester.edu/oil_supply_and_growth_submission.pdf⟩.

Schwartz, E and Smith, J E, Short-term variations and long-term dynamics in commodity prices. Management Science, 2000, pp. 893–911.

Schwartz, E S, The Stochastic Behavior of Commodity Prices: Implications for Valuation and Hedging. The Journal of Finance 52 1997:3.

Singleton, Kenneth J, Investor flows and the 2008 boom/bust in oil prices. Management Science, 60 2013:2, pp. 300–318 ⟨URL: http://pubsonline.informs.org/doi/abs/10.1287/mnsc.2013.1756⟩.

Stock, James H and Watson, Mark W, Forecasting inflation. Journal of Monetary Economics, 44 1999:2, pp. 293–335 ⟨URL: http://www.sciencedirect.com/science/article/pii/S0304393299000276⟩.

Szymanowska, Marta et al., An anatomy of commodity futures risk premia. Journal of Finance, forthcoming, 2013 ⟨URL: http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1343809⟩.

Trolle, A B and Schwartz, E S, Unspanned stochastic volatility and the pricing of commodity derivatives. Review of Financial Studies, 22 2009:11, pp. 4423–4461 ⟨URL: http://rfs.oxfordjournals.org/content/22/11/4423.short⟩.

Working, H, The Theory of Price of Storage. The American Economic Review, 39 1949:6, pp. 1254–1262 ⟨URL: http://www.jstor.org/stable/10.2307/1816601⟩.

Yogo, M, A consumption-based explanation of expected stock returns. Journal of Finance, 61 2006:2, pp. 539{\textendash}580.

A Robustness Checks

A.1 Alternative Measures of Real Activity

The predictability I find using the Chicago Fed National Activity Index also holds using other forward-looking measures of real activity. In this section I show that the same results obtain when I use the Aruba-Diebold-Scotti (ADS)[18] index (Aruoba, Diebold and Scotti, 2009) and the Conference Board's Leading Economic Index (LEI)[19] in place of the CFNAI.

The LEI is a weighted forward-looking index of real activity like the CFNAI, but uses different weights and macroeconomic time series. The ADS index is a real-time forward-looking index of real activity that is extracted by filtering from a third set of macroeconomic time series. The time series are similar because all three are intended as forward-looking measures of real activity, but they are not identical: the correlation between the ADS index and the CFNAI is 83.8% in levels and 58.7% in changes while the correlation between the LEI and the CFNAI is 8.6% in levels and 25.3% in changes.

Table 7 shows the results of the return forecasting regressions using the ADS index (Panel A) and the LEI (Panel B). We see that both alternative indices positively forecast oil futures returns, over and above the information in the futures curve.

Table 8 shows the feedback matrix $K_1^{\mathbb{P}}$ implied by estimating the affine model using the ADS index and the LEI in place of the CFNAI. As the CFNAI does, both the ADS index and the LEI forecast a higher spot price of oil (top right) and the spot price of oil negatively forecasts a lower value of the index (bottom left). Thus, the main conclusions are the same using alternative measures of real activity.

[18] https://www.philadelphiafed.org/research-and-data/real-time-center/business-conditions-index/
[19] https://www.conference-board.org/data/bcicountry.cfm?cid=1

Table 7: Panel A shows the results of forecasting returns to oil futures. r^2_{t+1} is the log return to the second nearby oil futures contract and $\overline{r}_{t+1}$ is the average log return to all active futures contracts with maturities from 2 to 12 months. The forecasting variables are 1) three sets of 'reduced-form' state variables $\mathcal{P}_t$ based on oil futures prices and 2) the Aruoba-Diebold-Scotti index ADS_t (Panel A) or the Leading Economic Index (LEI_t) plus log oil inventory INV_t. The data are monthly from from 1/1986 to 6/2014. Newey-West standard errors with lags are in parentheses.

Panel A: Aruoba-Diebold-Scotti (ADS) Index

$$r_{t+1} = \alpha + \beta_{ADS,INV} M_t + \beta_{\mathcal{P}} \mathcal{P}_t + \epsilon_{t+1}$$

	r^2_{t+1}			$\overline{r}_{t+1}$		
ADS_t	0.0317***	0.0295***	0.0293***	0.0247**	0.0239**	0.0238**
	(0.0109)	(0.0113)	(0.0116)	(0.0098)	(0.0101)	(0.0104)
INV_t	-0.002	-0.002	0.782	0.000	0.000	0.252
	(0.002)	(0.002)	(0.644)	(0.001)	(0.001)	(0.573)
Spanned Factors $\mathcal{P}_t$:	$PC^{1,2}$	PC^{1-5}	f^{1-12}	$PC^{1,2}$	PC^{1-5}	f^{1-12}
T	341	341	341	341	341	341
Adj. $R^2(\mathcal{P}_t)$	0.4%	0.7%	4.6%	0.2%	0.1%	3.9%
Adj. $R^2(\mathcal{P}_t + M_t)$	3.9%	3.6%	7.4%	3.5%	3.1%	6.7%
F-ratio	6.6***	5.5***	5.4***	6.3***	5.6***	5.5***

Panel B: Conference Board Leading Economic Index (LEI)

$$r_{t+1} = \alpha + \beta_{LEI,INV} M_t + \beta_{\mathcal{P}} \mathcal{P}_t + \epsilon_{t+1}$$

	r^2_{t+1}			$\overline{r}_{t+1}$		
LEI_t	0.1053*	0.1112*	0.1070*	0.0985**	0.1067**	0.1041**
	(0.0595)	(0.0569)	(0.0557)	(0.0467)	(0.0462)	(0.0451)
INV_t	−0.008**	−0.008**	1.177*	−0.005	−0.006*	0.571
	(0.004)	(0.003)	(0.605)	(0.003)	(0.003)	(0.531)
Spanned Factors $\mathcal{P}_t$:	$PC^{1,2}$	PC^{1-5}	f^{1-12}	$PC^{1,2}$	PC^{1-5}	f^{1-12}
T	341	341	341	341	341	341
Adj. $R^2(\mathcal{P}_t)$	0.4%	0.7%	4.6%	0.2%	0.1%	3.9%
Adj. $R^2(\mathcal{P}_t + M_t)$	1.4%	1.7%	5.4%	1.6%	1.6%	5.2%
F-ratio	2.1	2.1	1.9	2.9*	3.0*	2.8*

Table 8: Maximum likelihood (ML) estimates of the macro-finance model for Nymex crude oil futures, using data from 1/1986 to 6/2014. s, c are the spot price and annualized cost of carry respectively. ADS and LEI are the Aruoba-Diebold-Scotti index and the Conference Board Leading Economic Index respectively. INV is the log of the private U.S. crude oil inventory as reported by the EIA. The coefficients are over a monthly horizon, and the state variables are de-meaned. ML standard errors are in parentheses.

Panel A: Aruoba-Diebold-Scotti (ADS) Index

	$K_1^{\mathbb{P}}$		
	s_t	c_t	ADS_t
Δs_{t+1}	-0.004	0.059**	0.031***
	(0.008)	(0.027)	(0.009)
Δc_{t+1}	0.014*	−0.127***	−0.019**
	(0.008)	(0.025)	(0.009)
ΔADS_{t+1}	−0.069**	0.079	−0.264***
	(0.033)	(0.107)	(0.036)

Panel B: Conference Board Leading Economic Index (LEI)

	$K_1^{\mathbb{P}}$		
	s_t	c_t	LEI_t
Δs_{t+1}	−0.028**	0.061**	0.126**
	(0.011)	(0.027)	(0.054)
Δc_{t+1}	0.028***	−0.128***	-0.074
	(0.010)	(0.025)	(0.050)
ΔLEI_{t+1}	−0.002**	-0.001	0.003
	(0.001)	(0.002)	(0.003)

A.2 Excluding the Financial Crisis

Inspecting Figure 1, we question whether the results in the paper are driven by the huge swings in oil prices and real activity during 2008-2009. Table 9 presents the forecasting regressions estimated on a subsample from January 1986 to December 2007. We see that the conclusions are the same, and indeed the forecasting power of GRO is slightly stronger, when we omit 2008-2014.

Table 10 presents the full model estimated on the subsample from January 1986 to December 2007. The subsample estimate is similar to the full-sample estimate, and the key coefficients of ΔGRO_{t+1} on s_t and Δs_{t+1} on GRO_t remain statistically significant.

A.3 Time Varying Volatility

This section examines the results of the forecasting regressions in Table 1 when I add measures of time-varying volatility in oil futures. If volatility drives a higher hedge premium, then it might be an omitted factor that explains the positive association between real activity and the oil price forecast. I examine three standard volatility measures: $optvol_t$ is the implied volatility from short-term options on oil futures, $garchvol_t$ is the conditional volatility of Δf^1_{t+1} estimated as a GARCH(1,1) process, and $sqchg_t$ is the lagged squared change $(\Delta f^1_t)^2$ of the nearby log futures price.

Table 11 shows that the crude oil volatility indexes are indeed negatively correlated with GRO. However, time-varying volatility does not forecast oil prices or returns, and thus does not explain the forecasting power of real activity. Table 12 shows that none of the volatility factors is significant in the forecasting regressions, none of them significantly raises the adjusted R^2, and (most importantly) their inclusion does not alter the forecasting power of real activity.

Table 9: Panel A shows the results of forecasting returns to oil futures. r^2_{t+1} is the log return to the second nearby oil futures contract and $\overline{r}_{t+1}$ is the average log return to all active futures contracts with maturities from 2 to 12 months. Panel B shows the results of forecasting changes in the level factor PC^1. The forecasting variables are 1) three sets of 'reduced-form' state variables $\mathcal{P}_t$ based on oil futures prices and 2) the real activity index GRO_t and log oil inventory INV_t. The data are monthly from from 1/1986 to 12/2007. Newey-West standard errors with six lags are in parentheses.

Panel A: Forecasting Futures Returns

$$r_{t+1} = \alpha + \beta_{GRO,INV} M_t + \beta_{\mathcal{P}} \mathcal{P}_t + \epsilon_{t+1}$$

	r^2_{t+1}			$\overline{r}_{t+1}$		
GRO_t	0.0300***	0.0281***	0.0249**	0.0248***	0.0237***	0.0212***
	(0.0089)	(0.0092)	(0.0096)	(0.0067)	(0.0070)	(0.0072)
INV_t	-0.026	-0.022	-0.018	-0.007	-0.015	-0.012
	(0.120)	(0.115)	(0.104)	(0.087)	(0.080)	(0.074)
Spanned Factors $\mathcal{P}_t$:	$PC^{1,2}$	PC^{1-5}	f^{1-12}	$PC^{1,2}$	PC^{1-5}	f^{1-12}
T	263	263	263	263	263	263
Adj. $R^2(\mathcal{P}_t)$	-0.5%	-0.2%	5.6%	-0.3%	-0.1%	6.1%
Adj. $R^2(\mathcal{P}_t + M_t)$	2.3%	2.0%	7.1%	2.8%	2.6%	8.1%
F-ratio	4.7***	3.9**	3.1**	5.2***	4.6**	3.7**

Panel B: Forecasting PCs

$$\Delta PC_{t+1} = \alpha + \beta_{GRO,INV} M_t + \beta_{\mathcal{P}} \mathcal{P}_t + \epsilon_{t+1}$$

	ΔPC^1 (Change in level)			ΔPC^2 (Change in slope)		
GRO_t	0.0845***	0.0810***	0.0729***	0.0067	0.0067	0.0056
	(0.0227)	(0.0236)	(0.0243)	(0.0057)	(0.0057)	(0.0060)
INV_t	-0.014	-0.051	-0.039	-0.002	0.004	0.001
	(0.296)	(0.270)	(0.248)	(0.092)	(0.084)	(0.078)
Spanned Factors $\mathcal{P}_t$:	$PC^{1,2}$	PC^{1-5}	f^{1-12}	$PC^{1,2}$	PC^{1-5}	f^{1-12}
T	263	263	263	263	263	263
Adjusted $R^2(\mathcal{P}_t)$	-0.5%	-0.3%	5.6%	7.4%	8.5%	10.8%
Adj. $R^2(\mathcal{P}_t + M_t)$	2.7%	2.6%	7.7%	7.1%	8.3%	10.4%
F-ratio	5.2***	4.8***	3.8**	0.8	0.7	0.5

Table 10: Maximum likelihood (ML) estimate of the macro-finance model for Nymex crude oil futures using data from 1/1986 to 12/2007. s, c are the spot price and annualized cost of carry respectively. GRO is the monthly Chicago Fed National Activity Index. INV is the log of the private U.S. crude oil inventory as reported by the EIA. The coefficients are over a monthly horizon, and the state variables are de-meaned. ML standard errors are in parentheses.

	$K_0^{\mathbb{P}}$	$K_1^{\mathbb{P}}$			
		s_t	c_t	GRO_t	INV_t
s_{t+1}	0.011	-0.005	0.058	0.029***	-0.005
	(0.007)	(0.014)	(0.036)	(0.011)	(0.106)
c_{t+1}	-0.012	0.026*	−0.130***	-0.012	0.003
	(0.007)	(0.014)	(0.036)	(0.011)	(0.108)
GRO_{t+1}	0.057	−0.181**	0.651***	−0.580***	-1.653
	(0.035)	(0.070)	(0.178)	(0.054)	(0.526)
INV_{t+1}	0.003	−0.007*	0.033***	-0.003	−0.125***
	(0.002)	(0.004)	(0.009)	(0.003)	(0.027)

	$K_0^{\mathbb{Q}}$	$K_1^{\mathbb{Q}}$	
		s_t	c_t
s_{t+1}	-0.003	0.000	0.083***
	(0.007)	(0.005)	(0.011)
c_{t+1}	0.000	-0.009	−0.113***
	(0.012)	(0.014)	(0.031)

Shock Volatilities
[off-diagonal = % correlations]

	s	c	GRO	INV
s	0.102			
c	-84%	0.056		
GRO	7%	-1%	0.499	
INV	-20%	29%	2%	0.024

Table 11: The table shows the correlations of the monthly real activity index GRO and three indexes of time varying volatility in crude oil prices. The time series are monthly from 1/1989 to 6/2014 and have been demeaned. $garchvol_t$ is the conditional volatility of Δf^1_{t+1} estimated as a GARCH(1,1) process. $optvol_t$ is the implied volatility based on the prices of at-the-money options on one month futures. $sqchg_t$ is the squared change $(\Delta f^1_t)^2$ of the front-month futures contract last month.

	GRO_t	$sqchg_t$	$optvol_t$	$garchvol_t$
GRO_t	1			
$sqchg_t$	-24.8%	1		
$optvol_t$	-54.9%	50.7%	1	
$garchvol_t$	-51.8%	27.6%	68.7%	1

A.4 Year-on-Year Changes

Although futures returns are a stationary process (cf. Figure 5), they may contain slow-moving components i.e. time varying expected returns or regime shifts that are effectively nonstationary over a monthly horizon. Log futures prices f_t and the principal components portfolios $\mathcal{P}t$ that summarize them are themselves nonstationary or very close to it (cf. Figure 1). In this setting, forecasting regressions such as those presented in Table 1 may have poor small-sample properties (Ferson, Sarkissian and Simin, 2003).

To address this concern I rerun the regressions in Table 1 after transforming f_t and PC_t into year-on-year changes $f^{YOY}_t = f_t - f_{t-12}$, $PC^{YOY}_t = PC_t - PC_{t-12}$. The macro variables M_t need not be transformed as, first, they are clearly stationary (Figure 1) and, second, year-on-year differencing would eliminate the important variation in GRO (i.e. at business cycle frequency). Table 13 shows that after removing persistence in the regressors, the incremental forecasting power of real activity for futures returns and changes in the level factor is effectively unchanged.

Table 12: The table shows the results of forecasting returns to oil futures including measures of time-varying volatility. The data are monthly from 1/1986 to 6/2014 except *optvol* which is monthly from 1/1989 to 6/2014. r^2_{t+1} is the log excess return to the second nearby oil futures contract. $\overline{r}_{t+1}$ is the average log excess return to all active futures contracts with maturities up to 12 months. The forecasting variables are the Chicago Fed National Activity Index GRO_t, the first two PCs of log oil futures prices, and three measures of crude oil volatility. $optvol_t$ is the implied volatility based on the prices of at-the-money options on one month futures. $garchvol_t$ is the conditional volatility of Δf^1_{t+1} estimated as a GARCH(1,1) process. $sqchg_t$ is the lagged squared change $(\Delta f^1_t)^2$ of the log price of the first nearby futures contract. Newey-West standard errors with six lags are in parentheses.

$$r_{t+1} = \alpha + \beta_{GRO} M_t + \beta_{\mathcal{P}} PC^{1,2}_t + \beta_{VOL} VOL_t + \epsilon_{t+1}$$

	r^2_{t+1}			$\overline{r}_{t+1}$		
GRO_t	0.023**	0.027***	0.029**	0.019**	0.020***	0.023**
	(0.010)	(0.009)	(0.012)	(0.008)	(0.007)	(0.010)
$optvol_t$	-0.009			-0.004		
	(0.018)			(0.014)		
$garchvol_t$		-0.167			-0.240	
		(0.432)			(0.314)	
$sqchg_t$			0.002			0.003
			(0.008)			(0.006)
Spanned Factors $\mathcal{P}_t$:	$PC^{1,2}$	$PC^{1,2}$	$PC^{1,2}$	$PC^{1,2}$	$PC^{1,2}$	$PC^{1,2}$
T	295	341	341	295	341	341
Adj. $R^2(\mathcal{P}_t + GRO_t)$	4.1%	4.5%	4.5%	3.5%	3.7%	3.7%
Adj. $R^2(\mathcal{P}_t + GRO_t + VOL_t)$	3.9%	4.3%	4.2%	3.2%	3.7%	3.5%
F-ratio	0.4	0.4	0.1	0.1	1.2	0.3

Table 13: Panel A shows the results of forecasting returns to oil futures after transforming to year-on-year changes. r^2_{t+1} is the log return to the second nearby oil futures contract and $\overline{r}_{t+1}$ is the average log return to all active futures contracts with maturities from 2 to 12 months. Panel B shows the results of forecasting changes in the level factor PC^1. The forecasting variables are 1) three sets of year-on-year changes in 'reduced-form' state variables $\mathcal{P}^{YOY}_t$ based on oil futures prices and 2) the real activity index GRO_t and log oil inventory INV_t. The data are monthly from from 1/1986 to 6/2014. Newey-West standard errors with six lags are in parentheses.

Panel A: Forecasting Futures Returns

$$r_{t+1} = \alpha + \beta_{GRO,INV} M_t + \beta_{\mathcal{P}} \mathcal{P}^{YOY}_t + \epsilon_{t+1}$$

	r^2_{t+1}			$\overline{r}_{t+1}$		
GRO_t	0.0283***	0.0277**	0.0271**	0.0196**	0.0197**	0.0200*
	(0.0109)	(0.0112)	(0.0114)	(0.0094)	(0.0098)	(0.0099)
INV_t	-0.039	-0.048	-0.050	-0.038	-0.046	-0.047
	(0.079)	(0.072)	(0.070)	(0.062)	(0.057)	(0.055)
Spanned Factors $\mathcal{P}^{YOY}_t$:	$PC^{1,2}$	PC^{1-5}	f^{1-12}	$PC^{1,2}$	PC^{1-5}	f^{1-12}
T	329	329	329	329	329	329
Adj. $R^2(\mathcal{P}_t)$	2.4%	2.2%	1.2%	2.0%	1.4%	-0.2%
Adj. $R^2(\mathcal{P}_t + M_t)$	6.5%	5.9%	4.6%	5.0%	4.4%	2.8%
F-ratio	8.0***	7.4***	6.7***	6.2***	6.0***	5.9***

Panel B: Forecasting PCs

$$\Delta PC_{t+1} = \alpha + \beta_{GRO,INV} M_t + \beta_{\mathcal{P}} \mathcal{P}^{YOY}_t + \epsilon_{t+1}$$

	ΔPC^1 (Change in level)			ΔPC^2 (Change in slope)		
GRO_t	0.095**	0.091**	0.088*	0.010	0.009	0.008
	(0.043)	(0.045)	(0.046)	(0.006)	(0.007)	(0.007)
INV_t	-0.231	-0.270	-0.287	0.051	0.034	0.030
	(0.281)	(0.254)	(0.251)	(0.060)	(0.056)	(0.055)
Spanned Factors $\mathcal{P}_t$:	$PC^{1,2}$	PC^{1-5}	f^{1-12}	$PC^{1,2}$	PC^{1-5}	f^{1-12}
T	329	329	329	329	329	329
Adjusted $R^2(\mathcal{P}_t)$	5.7%	5.7%	7.7%	9.0%	9.5%	11.5%
Adj. $R^2(\mathcal{P}_t + M_t)$	8.4%	8.1%	10.0%	9.4%	9.6%	11.5%
F-ratio	5.8***	5.2***	4.9**	0.8	0.7	0.5

B Real Option Valuation

I model the log lifting cost (per-barrel cost of extraction) as

$$l_t = \kappa_l + 0.1 s_t + 0.01 GRO_t + \epsilon_t^l,\ \epsilon_t^l \sim N(0, \sigma_l)$$

That is, l_t varies with both s_t and GRO_t as well as having an i.i.d. idiosyncratic component with volatility σ_l. The other parameters in the simulated data are in Table 14. Notice the third row of $K_1^{\mathbb{Q}}$, which was not present in the model estimates. Pricing assets with payoffs that depend on M_t requires the risk neutral dynamics of M_t. In principle one could estimate the risk neutral dynamics of M_t with a tracking portfolio for GRO (e.g. Lamont (2001)), but for simplicity I assume that exposure to GRO carries a fixed risk premium of λ.

I compute option values for different starting values of the lifting cost $L_0 = exp(l_0)$, with $S_0 = exp(s_0)$ equal to \$80 per barrel and $c_0 = 0$. This simulates an oil firm evaluating wells that differ in their current lifting cost, conditional on a spot price of \$80 and a flat futures curve.

Internet Appendix

Unspanned Macroeconomic Factors in Oil Futures

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1 Model Specification and Risk Premiums

Consider a Gaussian model where the log spot price s_t of a commodity is a function of N_L spanned state variables L_t, which may be latent or observed, and N_M unspanned state variables M_t that are observed:

$$\begin{aligned} \begin{bmatrix} L_{t+1} \\ M_{t+1} \end{bmatrix} &= K_{0X}^{\mathbb{P}} + K_{1X}^{\mathbb{P}} X_t + \Sigma_X \epsilon_{t+1}^{\mathbb{P}} \\ L_{t+1} &= K_{0L}^{\mathbb{Q}} + K_{1L}^{\mathbb{Q}} L_t + \Sigma_L \epsilon_{t+1}^{\mathbb{Q}} \\ s_t &= \delta_0 + \delta_1' L_t \end{aligned} \tag{1}$$

where

- $\mathbb{P}$ denotes dynamics under the historical or data generating measure
- $\mathbb{Q}$ denotes dynamics under the risk neutral measure
- $\epsilon_{L,t+1}^{\mathbb{Q}} \sim N(0,\, I_{N_L})$, $\epsilon_{t+1}^{\mathbb{P}} \sim N(0,\, I_N)$
- Σ_L is the top left $N_L \times N_L$ block of Σ_X; Σ_L, Σ_X are lower triangular

(1) is equivalent to specifying the equation for s_t and the $\mathbb{P}$-dynamics plus a lognormal affine discount factor with 'essentially affine' prices of risk as in Duffee (2002). For $N_M = 0$ the framework includes models such as Gibson and Schwartz (1990); Schwartz (1997); Schwartz and Smith (2000) as special cases (see Appendix 4). Standard recursions show that (1) implies affine log prices for futures,

$$f_t = A + BX_t \tag{2}$$

$$f_t = \begin{bmatrix} f_t^1 & f_t^2 & \dots & f_t^J \end{bmatrix}'$$

where f_t^j is the price of a j period future and J is the number of futures with different maturities.

Estimating the model as written presents difficulties; with two latent factors and two macro factors there are 40 free parameters. Different sets of parameter values are observationally equivalent due to rotational indeterminacy of the latent factors. Discussing models of the form (1) for bond yields, Hamilton and Wu (2012) refer to *"tremendous numerical challenges in estimating the necessary parameters from the data due to highly nonlinear and badly behaved likelihood surfaces."* In general, affine models for futures identify the model by specifying dynamics that are less general than (1).

Joslin, Priebsch and Singleton (2014) note that if N_L linear combinations of bond yields are measured without error, then any model of yields of the form (1) implies a model with observable factors in place of the latent factors. They construct a minimal parametrization where no sets of parameters are redundant - models in the "JPS form" are unique. Thus the likelihood surface is well behaved and contains a single global maximum. Their results hold to a very close approximation if the linear combinations of yields are observed with relatively small and idiosyncratic errors.

Section 2 demonstrates the same result for futures pricing: if N_L linear combinations of log futures prices are measured without error,

$$\mathcal{P}_t = W f_t \tag{3}$$

for any full rank $N_L \times J$ matrix W, then any model of the form (1) is observationally equivalent to a unique model of the form

$$\begin{bmatrix} \Delta\mathcal{P}_{t+1} \\ \Delta UM_{t+1} \end{bmatrix} = \begin{aligned} \Delta Z_{t+1} &= K_0^{\mathbb{P}} + K_1^{\mathbb{P}} Z_t + \Sigma_Z \epsilon_{t+1}^{\mathbb{P}} \\ \Delta\mathcal{P}_{t+1} &= K_0^{\mathbb{Q}} + K_1^{\mathbb{Q}}\mathcal{P}_t + \Sigma_{\mathcal{P}}\epsilon_{t+1}^{\mathbb{Q}} \\ s_t &= \rho_0 + \rho_1\mathcal{P}_t \end{aligned} \tag{4}$$

parametrized by $\theta = (\lambda^{\mathbb{Q}},\ p_\infty,\ \Sigma_Z,\ K_0^{\mathbb{P}},\ K_1^{\mathbb{P}})$, where

- λ^Q are the N_L ordered eigenvalues of $K_1^{\mathbb{Q}}$
- p_∞ is a scalar intercept
- Σ_Z is the lower triangular Cholesky decomposition of the covariance matrix of innovations in the state variables
- $\Sigma_{\mathcal{P}}\Sigma_{\mathcal{P}}' = [\Sigma_Z\Sigma_Z']_{N_L}$, the top left $N_L \times N_L$ block of $\Sigma_Z\Sigma_Z'$

1.1 $\mathcal{P}_t$ Measured Without Error

In this paper I assume that while each of the log futures maturities is observed with iid measurement error, the pricing factors $\mathcal{P}_t^1$ and $\mathcal{P}_t^2$ are measured without error.

$$f_t^j = A_j + B_j\mathcal{P}_t + \nu_t^j,\ \nu_t^j \sim N(0, \zeta_j^2)$$

The use of the first two PCs of log price levels is not important: in unreported results I find that all estimates and results are effectively identical using other alternatives such as the first two PCs of log price changes or of returns, or a priori weights such as

$$W = \begin{bmatrix} 1 & ... & 1 \\ 0 & ... & 12 \end{bmatrix}$$

The identifying assumption that N_L linear combinations of yields are measured without error is common in the bond yields literature beginning with Chen and Scott (1993). Given the model parameters, values of the latent factors at each date are then extracted by inverting the relation (2). The same assumption is used to identify previous latent factor models for commodity futures (see Gibson and Schwartz (1990); Casassus and Collin-Dufresne (2005); Hamilton and Wu (2014)). In unreported results I find that all estimates and results are effectively identical if the pricing factors are estimated via the Kalman filter.

1.2 Rotating to s_t and c_t

Once the model is estimated in the JPS form, I rotate $(\mathcal{P}_t^1, \mathcal{P}_t^2)$ to be the model implied log spot price and instantaneous cost of carry, $(s_t,\, c_t)$. For s_t this is immediate:

$$s_t = \rho_0 + \rho_1 \mathcal{P}_t$$

For c_t the definition is as follows. Any agent with access to a storage technology can buy the spot commodity, sell a one month future, store for one month and make delivery. Add up all the costs and benefits of doing so (including interest, costs of storage, and convenience yield) and express them as quantity c_t where the total cost in dollar terms $= S_t(e^{c_t} - 1)$.

Then in the absence of arbitrage it must be the case that

$$F_t^1 = S_t e^{c_t}$$

$$f_t^1 = s_t + c_t = E^{\mathbb{Q}}\left[s_{t+1}\right] + \frac{1}{2}\sigma_s^2$$

$$c_t = E^{\mathbb{Q}}\left[\Delta s_{t+1}\right] + \frac{1}{2}\sigma_s^2$$

$$= \rho_1[K_0^{\mathbb{Q}} + K_1^{\mathbb{Q}}\mathcal{P}_t] + \frac{1}{2}\sigma_s^2$$

1.3 Risk premiums and s_t, c_t:

Szymanowska et al. (2013) define the per-period log basis $y_t^n \equiv f_t^n - s_t$, and define two risk premiums based on different futures trading strategies; the spot premium $\pi_{s,t}$ and the term premium $\pi_{y,t}^n$.

The spot premium is defined as

$$\pi_{s,t} \equiv E_t\left[s_{t+1} - s_t\right] - y_t^1$$

$$= E_t\left[s_{t+1}\right] - f_t^1 = E_t^{\mathbb{P}}\left[s_{t+1}\right] - E_t^{\mathbb{Q}}\left[s_{t+1}\right] - \frac{1}{2}\sigma_s^2$$

$$\Rightarrow \pi_{s,t} = \Lambda_{s_t} - \frac{1}{2}\sigma_s^2$$

The term premium is defined as

$$\pi_{y,t}^n \equiv y_t^1 + (n-1)E_t\left[y_{t+1}^{n-1}\right] - ny_t^n$$

$$= f_t^1 + (n-1)E_t\left[f_{t+1}^{n-1}\right] - nf_t^n$$

The one month term premium is always zero, because storing for one month is riskless.

$$\pi_{y,t}^{(1)} = f_t^1 + 0 - f_t^1 = 0$$

$$\pi_{y,t}^{(2)} = c_t + E_t^{\mathbb{P}}\left[c_{t+1}\right] - 2E_t^{\mathbb{Q}}\left[s_{t+2} - s_{t+1} + s_{t+1} - s_t\right]$$

$$= E_t^{\mathbb{P}}\left[c_{t+1}\right] - E_t^{\mathbb{Q}}\left[c_{t+1}\right] - E_t^{\mathbb{Q}}\left[s_{t+2} - s_{t+1}\right] - c_t$$

$$\Rightarrow \pi_{y,t}^{(2)} = \Lambda_{c_t} - \left(c_t + E_t^{\mathbb{Q}}\left[c_{t+1}\right]\right)$$

Thus the spot premium and term premium of Szymanowska et al. (2013) each have a natural expression in our affine framework. The spot premium is exactly the risk premium attached to shocks to the log spot price s_t plus a small constant. The term premium is the risk premium attached to shocks to the cost of carry minus the (risk-neutral) total expected cost of carry.

2 JPS Parametrization

I assume that N_L linear combinations of log futures prices are measured without error,

$$\mathcal{P}_t^L = W f_t$$

for any full-rank real valued $N_L \times J$ matrix W, and show that any model of the form

$$\begin{array}{rcl} \begin{bmatrix} \Delta L_{t+1} \\ \Delta M_{t+1} \end{bmatrix} = \Delta X_{t+1} & = & K_{0X}^{\mathbb{P}} + K_{1X}^{\mathbb{P}} X_t + \Sigma_X \epsilon_{t+1}^{\mathbb{P}} \\ \Delta L_{t+1} & = & K_{0L}^{\mathbb{Q}} + K_{1L}^{\mathbb{Q}} X_t + \Sigma_L \epsilon_{L,t+1}^{\mathbb{Q}} \\ s_t & = & \delta_0 + \delta_1' X_t \end{array} \tag{5}$$

is observationally equivalent to a unique model of the form

$$\begin{array}{rcl} \begin{bmatrix} \Delta \mathcal{P}_{t+1}^L \\ \Delta M_{t+1} \end{bmatrix} = \Delta Z_{t+1} & = & K_0^{\mathbb{P}} + K_1^{\mathbb{P}} Z_t + \Sigma_Z \epsilon_{Z,t+1}^{\mathbb{P}} \\ \Delta \mathcal{P}_{t+1}^L & = & K_0^{\mathbb{Q}} + K_1^{\mathbb{Q}} Z_t + \Sigma_{\mathcal{P}} \epsilon_{t+1}^{\mathbb{Q}} \\ s_t & = & \rho_0 + \rho_1' Z_t \end{array} \tag{6}$$

which is parametrized by $\theta = (\lambda^{\mathbb{Q}},\ p_\infty,\ \Sigma_Z,\ K_0^{\mathbb{P}},\ K_1^{\mathbb{P}})$.

The proof that follows is essentially the same as that of Joslin, Priebsch and Singleton (2014). Joslin, Singleton and Zhu (2011) demonstrates the result for all cases including zero, repeated and complex eigenvalues.

Assume the model (5) under consideration is nonredundant, that is, there is no observationally equivalent model with fewer than N state variables. If there is such a model, switch to it and proceed.

2.1 Observational Equivalence

Given any model of the form (5), the $J \times 1$ vector of log futures prices f_t is affine in L_t,

$$f_t = A_L + B_L L_t$$

Hence the set of N_L linear combinations of futures prices, $\mathcal{P}_t^L$, is as well:

$$\mathcal{P}_t^L = W_L f_t = W_L A_L + W_L B_L L_t$$

Assume that the N_L ordered elements of $\lambda^{\mathbb{Q}}$, the eigenvalues of $K_{1L}^{\mathbb{Q}}$, are real, distinct and nonzero. There exists a matrix C such that $K_{1L}^{\mathbb{Q}} = C\text{diag}(\lambda^{\mathbb{Q}})C^{-1}$. Define $D = Cdiag(\delta_1)C^{-1}$, $D^{-1} = Cdiag(\delta_1)^{-1}C^{-1}$ and

$$Y_t = D[L_t + \left(K_{1L}^{\mathbb{Q}}\right)^{-1} K_{0L}^{\mathbb{Q}}]$$

$$\Rightarrow \; L_t = D^{-1}Y_t - \left(K_{1L}^{\mathbb{Q}}\right)^{-1} K_{0L}^{\mathbb{Q}}$$

Then

$$\Delta Y_{t+1} = D\Delta L_{t+1}$$

$$= D[K_{0L}^{\mathbb{Q}} + K_{1L}^{\mathbb{Q}}(D^{-1}Y_t - \left(K_{1L}^{\mathbb{Q}}\right)^{-1} K_{0L}^{\mathbb{Q}}) + \Sigma_L \epsilon_{L,t+1}^{\mathbb{Q}}]$$

$$= \text{diag}(\lambda^{\mathbb{Q}})Y_t + D\Sigma_L \epsilon_{L,t+1}^{\mathbb{Q}}$$

and

$$\begin{bmatrix} \Delta Y_{t+1} \\ \Delta M_{t+1} \end{bmatrix} = \begin{bmatrix} D & 0 \\ 0 & I_{\mathcal{M}} \end{bmatrix} [K_{0X}^{\mathbb{P}} + K_{1X}^{\mathbb{P}} (\begin{bmatrix} D^{-1} & 0 \\ 0 & I_{\mathcal{M}} \end{bmatrix} \begin{bmatrix} Y_t \\ M_t \end{bmatrix} - \begin{bmatrix} \left(K_{1L}^{\mathbb{Q}}\right)^{-1} K_{0L}^{\mathbb{Q}} \\ 0 \end{bmatrix}) + \Sigma_X \epsilon_{t+1}^{\mathbb{P}}]$$

$$= K_{0Y}^{\mathbb{P}} + K_{1Y}^{\mathbb{P}} \begin{bmatrix} Y_t \\ M_t \end{bmatrix} + \begin{bmatrix} D & 0 \\ 0 & I_{\mathcal{M}} \end{bmatrix} \Sigma_X \epsilon_{t+1}^{\mathbb{P}}$$

and

$$p_t = \delta_0 + \delta_1' L_t = \delta_0 + \delta_1' D^{-1} Y_t - \delta_1' \left(K_{1L}^{\mathbb{Q}}\right)^{-1} K_{0L}^{\mathbb{Q}} = p_\infty + \iota \cdot Y_t$$

where ι is a row of N_L ones.

$$f_t = A_Y + B_Y Y_t$$

$$\mathcal{P}_t^L = W f_t = W A_Y + W B_Y Y_t$$

The model is nonredundant $\Rightarrow$ WB_Y is invertible:

$$Y_t = (WB_Y)^{-1} \mathcal{P}_t^L - (WB_Y)^{-1} W A_Y$$

$$\cdot \mathcal{P}_{t+1}^L = WB_Y \Delta Y_{t+1} = WB_Y \mathrm{diag}(\lambda^{\mathbb{Q}}) [(WB_Y)^{-1} \mathcal{P}_t^L - (WB_Y)^{-1} W A_Y] + WB_Y D \Sigma_L \epsilon_{L,t+1}^{\mathbb{Q}}$$

$$= K_0^{\mathbb{Q}} + K_1^{\mathbb{Q}} \mathcal{P}_t^L + \Sigma_{\mathcal{P}} \epsilon_{t+1}^{\mathbb{Q}}$$

Further,

$$\Delta Z_{t+1} = \begin{bmatrix} \cdot\mathcal{P}^L_{t+1} \\ \Delta M_{t+1} \end{bmatrix} = \begin{bmatrix} WB_Y & 0 \\ 0 & I_{\mathcal{M}} \end{bmatrix} \begin{bmatrix} \Delta Y_{t+1} \\ \Delta M_{t+1} \end{bmatrix}$$

$$= \begin{bmatrix} WB_Y & 0 \\ 0 & I_{\mathcal{M}} \end{bmatrix} \left(K^{\mathbb{P}}_{0Y} + K^{\mathbb{P}}_{1Y} \begin{bmatrix} Y_t \\ M_t \end{bmatrix} + \begin{bmatrix} D & 0 \\ 0 & I_{\mathcal{M}} \end{bmatrix} \Sigma_X \epsilon^{\mathbb{P}}_{t+1} \right)$$

$$= K^{\mathbb{P}}_0 + K^{\mathbb{P}}_1 Z_t + \Sigma_Z \epsilon^{\mathbb{P}}_{t+1}$$

$$p_t = p_\infty + \iota \cdot Y_t = p_\infty + \iota \cdot (WB_Y)^{-1}\mathcal{P}^L_t - \iota \cdot (WB_Y)^{-1} W A_Y = \rho_0 + \rho_1' \mathcal{P}^L_t$$

QED. Collecting the formulas: given any model of the form (1), there is an observationally equivalent model of the form (4), parametrized by $\theta = (\lambda^{\mathbb{Q}},\ p_\infty,\ \Sigma_Z,\ K^{\mathbb{P}}_0,\ K^{\mathbb{P}}_1)$, where

- $D = Cdiag(\delta_1)^{-1}C^{-1}$
- $\Sigma_Z = \begin{bmatrix} WB_Y D & 0 \\ 0 & I_{\mathcal{M}} \end{bmatrix} \Sigma_X,\ \Sigma_{\mathcal{P}} = [\Sigma_Z]_{\mathcal{L}\mathcal{L}}$
- $B_Y = \begin{bmatrix} \iota'[I_{\mathcal{L}+\mathcal{M}} + \text{diag}(\lambda^{\mathbb{Q}})] \\ \vdots \\ \iota'[I_{\mathcal{L}+\mathcal{M}} + \text{diag}(\lambda^{\mathbb{Q}})]^J \end{bmatrix}$
- $A_Y = \begin{bmatrix} p_\infty + \frac{1}{2}\iota'\Sigma_{\mathcal{P}}\Sigma_{\mathcal{P}}'\iota \\ \vdots \\ A_{Y,J-1} + \frac{1}{2} B_{Y,J-1}\Sigma_{\mathcal{P}}\Sigma_{\mathcal{P}}' B_{Y,J-1}' \end{bmatrix}$
- $K^{\mathbb{Q}}_1 = WB_Y \text{diag}(\lambda^{\mathbb{Q}})(WB_Y)^{-1},\ K^{\mathbb{Q}}_0 = -K^{\mathbb{Q}}_1 W A_Y$
- $\rho_0 = p_\infty - \iota \cdot (WB_Y)^{-1} W A_Y,\ \rho_1' = \iota \cdot (WB_Y)^{-1}$

In estimation I adopt the alternate form

- $\Delta Y_{t+1} = \begin{bmatrix} p_\infty \\ 0 \end{bmatrix} + \mathrm{diag}(\lambda^{\mathbb{Q}}) Y_t + D\Sigma_X \epsilon^{\mathbb{Q}}_{t+1}$
- $p_t = \iota \cdot Y_t$
- $A_Y = \begin{bmatrix} p_\infty + \frac{1}{2}\iota' \Sigma_{\mathcal{P}} \Sigma'_{\mathcal{P}} \iota \\ \vdots \\ A_{Y,J-1} + B_{Y,J-1} \begin{bmatrix} p_\infty \\ 0 \end{bmatrix} + \frac{1}{2} B_{Y,J-1} \Sigma_{\mathcal{P}} \Sigma'_{\mathcal{P}} B'_{Y,J-1} \end{bmatrix}$
- $K_1^{\mathbb{Q}} = W B_Y \mathrm{diag}(\lambda^{\mathbb{Q}})(W B_Y)^{-1}$, $K_0^{\mathbb{Q}} = W B_Y \begin{bmatrix} p_\infty \\ 0 \end{bmatrix} - K_1^{\mathbb{Q}} W A_Y$
- $\rho_0 = -\iota \cdot (W B_Y)^{-1} W A_Y$, $\rho_1' = \iota \cdot (W B_Y)^{-1}$

which is numerically stable when $\lambda^{\mathbb{Q}}(1) \to 0$. See the online supplement to JSZ 2011.

2.2 Uniqueness

We consider two models of the form (4) with parameters θ and $\hat{\theta} = (\hat{\lambda}^{\mathbb{Q}},\ \hat{p}_\infty,\ \hat{\Sigma}_Z,\ \hat{K}_0^{\mathbb{P}},\ \hat{K}_1^{\mathbb{P}})$ that are observationally equivalent and show that this implies $\theta = \hat{\theta}$.

Since $Z_t = \begin{bmatrix} \mathcal{P}_t^L \\ M_t \end{bmatrix}$ are all observed, $\{\Sigma_Z,\ K_0^{\mathbb{P}},\ K_1^{\mathbb{P}}\} = \{\hat{\Sigma}_Z,\ \hat{K}_0^{\mathbb{P}},\ \hat{K}_1^{\mathbb{P}}\}$.

Since $f_t = A + BZ_t$ are observed, $A(\theta) = A(\hat{\theta})$, $B(\theta) = B(\hat{\theta})$.

Suppose $\lambda^{\mathbb{Q}} \neq \hat{\lambda}^{\mathbb{Q}}$. Then by the uniqueness of the ordered eigenvalue decomposition,

$$B_Y^j(\lambda) \neq B_Y^j(\hat{\lambda})\, \forall j$$

$$\Rightarrow W B_Y(\lambda) \neq W B_Y(\hat{\lambda}) \ \Rightarrow (W B_Y(\lambda))^{-1} \neq (W B_Y(\hat{\lambda}))^{-1}$$

$$\Rightarrow \rho_1(\lambda) \neq \rho_1(\hat{\lambda}) \Rightarrow B(\lambda) \neq B(\hat{\lambda})$$

, a contradiction. Hence $\lambda^{\mathbb{Q}} = \hat{\lambda}^{\mathbb{Q}}$. Then $A(\lambda^{\mathbb{Q}}, p^{\infty}) = A(\hat{\lambda}^{\mathbb{Q}}, \hat{p}^{\infty}) \Rightarrow p^{\infty} = \hat{p}^{\infty}$.

3 Estimation

Given the futures prices and macroeconomic time series $\{f_t, M_t\}_{t=1,\dots,T}$ and the set of portfolio weights W that define the pricing factors:

$$\mathcal{P}_t = W f_t$$

we need to estimate the minimal parameters $\theta = (\lambda^{\mathbb{Q}}, p_{\infty}, \Sigma_Z, K_0^{\mathbb{P}}, K_1^{\mathbb{P}})$ in the JPS form. The estimation is carried out by maximum likelihood (ML). If no restrictions are imposed (i.e. we are estimating the canonical model (5)), then $K_0^{\mathbb{P}}, K_1^{\mathbb{P}}$ do not affect futures pricing and are estimated consistently via OLS. Otherwise $K_0^{\mathbb{P}}, K_1^{\mathbb{P}}$ are obtained by GLS taking the restrictions into account. The OLS estimate of Σ_Z is used as a starting value, and the starting value for p_{∞} is the unconditional average of the nearest-maturity log futures price. Both were always close to their ML value. Finally I search over a range of values for the eigenvalues $\lambda^{\mathbb{Q}}$.

After the ML estimate of the model in the JPS form is found, I rotate and translate the spanned factors from $\mathcal{P}_t^1, \mathcal{P}_t^2$ to s_t, c_t as described in 1.2. I rotate and translate UM_t to M_t, so that the estimate reflects the behavior of the time series M_t:

$$\begin{bmatrix} s_t \\ c_t \\ M_t \end{bmatrix} = \begin{bmatrix} \rho_0 \\ \frac{1}{2}\sigma_s^2 + \rho_1 K_0^{\mathbb{Q}} \\ \alpha_{M\mathcal{P}} \end{bmatrix} + \begin{bmatrix} \rho_1 & 0_{1\times N_M} \\ \rho_1 K_1^{\mathbb{Q}} & 0_{1\times N_M} \\ 0_{N_M\times 1} & \beta_{M\mathcal{P}} \end{bmatrix} \begin{bmatrix} \mathcal{P}_t \\ UM_t \end{bmatrix}$$

where

$$M_t = \alpha_{M\mathcal{P}} + \beta_{M\mathcal{P}}\mathcal{P}_t + UM_t$$

4 Comparison with other Futures Models

The model (1) is a canonical affine Gaussian model, so any affine Gaussian model is a special case. For example, the Gibson and Schwartz (1990); Schwartz (1997); Schwartz and Smith (2000) two factor model in discrete time is the following:

$$\begin{bmatrix} \Delta s_{t+1} \\ \Delta \delta_{t+1} \end{bmatrix} = \begin{bmatrix} \mu \\ \kappa\alpha \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ 0 & -\kappa \end{bmatrix} \begin{bmatrix} s_t \\ \delta_t \end{bmatrix} + \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix} \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}^{1/2} \epsilon^{\mathbb{P}}_{t+1} \tag{7}$$

$$\begin{bmatrix} \Delta s_{t+1} \\ \Delta \delta_{t+1} \end{bmatrix} = \begin{bmatrix} r \\ \kappa\alpha - \lambda \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ 0 & -\kappa \end{bmatrix} \begin{bmatrix} s_t \\ \delta_t \end{bmatrix} + \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix} \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}^{1/2} \epsilon^{\mathbb{Q}}_{t+1} \tag{8}$$

which is clearly a special case of (1).

The Casassus and Collin-Dufresne (2005) model in discrete time is:

$$\begin{bmatrix} \Delta X_{t+1} \\ \Delta \hat{\delta}_{t+1} \\ \Delta r_{t+1} \end{bmatrix} = \begin{bmatrix} \kappa_X^P \theta_X^P + \kappa_{Xr}^P \theta_r^P + \kappa_{X\hat{\delta}}^P \theta_{\hat{\delta}}^P \\ \kappa_{\hat{\delta}}^P \theta_{\hat{\delta}}^P \\ \kappa_r^P \theta_r^P \end{bmatrix} + \begin{bmatrix} -\kappa_X^P & -\kappa_{X\hat{\delta}}^P & -\kappa_{Xr}^P \\ 0 & -\kappa_{\hat{\delta}}^P & 0 \\ 0 & 0 & -\kappa_r^P \end{bmatrix} \begin{bmatrix} X_t \\ \hat{\delta}_t \\ r_t \end{bmatrix} + \begin{bmatrix} \sigma_X & 0 & 0 \\ 0 & \sigma_{\hat{\delta}} & 0 \\ 0 & 0 & \sigma_r \end{bmatrix} \begin{bmatrix} 1 & & \\ \rho_{X\delta} & 1 & \\ \rho_{Xr} & \rho_{\delta r} & 1 \end{bmatrix}^{1/2} \epsilon^{\mathbb{P}}_{t+1} \tag{9}$$

$$\begin{bmatrix} \Delta X_{t+1} \\ \Delta \hat{\delta}_{t+1} \\ \Delta r_{t+1} \end{bmatrix} = \begin{bmatrix} \alpha_X \theta_X^Q + (\alpha_r - 1)\theta_r^Q + \theta_{\hat{\delta}}^Q \\ \kappa_{\hat{\delta}}^Q \theta_{\hat{\delta}}^Q \\ \kappa_r^Q \theta_r^Q \end{bmatrix} + \begin{bmatrix} -\alpha_X & -1 & 1-\alpha_r \\ 0 & -\kappa_{\hat{\delta}}^Q & 0 \\ 0 & 0 & -\kappa_r^Q \end{bmatrix} \begin{bmatrix} X_t \\ \hat{\delta}_t \\ r_t \end{bmatrix} + \begin{bmatrix} \sigma_X & 0 & 0 \\ 0 & \sigma_{\hat{\delta}} & 0 \\ 0 & 0 & \sigma_r \end{bmatrix} \begin{bmatrix} 1 & & \\ \rho_{X\delta} & 1 & \\ \rho_{Xr} & \rho_{\delta r} & 1 \end{bmatrix}^{1/2} \epsilon^{\mathbb{Q}}_{t+1} \tag{10}$$

(see their formulas 7, 12, 13 and 27, 28, 30) which is the Schwartz three factor model with more flexible risk premiums.

References

Casassus, J. and Collin-Dufresne, P., Stochastic Convenience Yield implied from Commodity Futures and Interest Rates. The Journal of Finance, 60 (5) 2005, pp. 2283–2331 ⟨URL: http://onlinelibrary.wiley.com/doi/10.1111/j.1540-6261.2005.00799.x/abstract⟩.

Chen, R. R. and Scott, L., Maximum likelihood estimation for a multifactor equilibrium model of the term structure of interest rates. The Journal of Fixed Income, 3 1993:3, pp. 14–31 ⟨URL: http://www.iijournals.com/doi/abs/10.3905/jfi.1993.408090⟩.

Duffee, G. R., Term premia and interest rate forecasts in affine models. The Journal of Finance, 57 2002:1, pp. 405–443 ⟨URL: http://onlinelibrary.wiley.com/doi/10.1111/1540-6261.00426/abstract⟩.

Gibson, R. and Schwartz, E. S., Stochastic convenience yield and the pricing of oil contingent claims. Journal of Finance, 1990, pp. 959–976 ⟨URL: http://www.jstor.org/stable/10.2307/2328801⟩.

Hamilton, J. D. and Wu, J. C., Identification and Estimation of Gaussian Affine Term Structure Models. Journal of Econometrics, 2012 ⟨URL: http://www.sciencedirect.com/science/article/pii/S0304407612000450⟩.

Hamilton, J.D. and Wu, J. C., Risk Premia in Crude Oil Futures Prices. Journal of International Money and Finance, 42 2014, pp. 9–37.

Joslin, S., Priebsch, M. and Singleton, K.J., Risk Premiums in Dynamic Term Structure Models with Unspanned Macro Risks. Journal of Finance, 69 2014, pp. 1197–1233 ⟨URL: http://onlinelibrary.wiley.com/doi/10.1111/jofi.12131/abstract⟩.

Joslin, Scott, Singleton, Kenneth J. and Zhu, Haoxiang, A New Perspective on Gaussian Dynamic Term Structure Models. Review of Financial Studies, 24 March 2011:3, pp. 926 –970 ⟨URL: http://www-bcf.usc.edu/~sjoslin/papers/JSZ_RFS_2011.pdf⟩.

Schwartz, E. and Smith, J.E., Short-term variations and long-term dynamics in commodity prices. Management Science, 2000, pp. 893–911.

Schwartz, E.S., The Stochastic Behavior of Commodity Prices: Implications for Valuation and Hedging. The Journal of Finance 52 1997:3.

Szymanowska, Marta et al., An anatomy of commodity futures risk premia. Journal of Finance, forthcoming, 2013 ⟨URL: http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1343809⟩.

Table 14: Parameters of the calibration for computing real option values

	$K_0^{\mathbb{P}}$	$K_1^{\mathbb{P}}$		
		s_t	c_t	GRO_t
s_{t+1}	0.00	1.00	0.083	0.03
c_{t+1}	0.00	0.00	0.90	0.00
GRO_{t+1}	0.00	-0.10	0.00	0.60

Σ	s	c	GRO
s	0.10		
c	-0.08	0.06	
GRO	0	0	0.50

	$K_0^{\mathbb{Q}}$	$K_1^{\mathbb{Q}}$		
		s_t	c_t	GRO_t
s_{t+1}	0.00	1.00	0.083	0.00
c_{t+1}	0.00	0.00	0.90	0.00
GRO_{t+1}	$-\lambda$	-0.10	0.00	0.60